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First Part

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Robinson's Shorter Course.

THE

COMPLETE
ARITHMETIC.

ORAL AND WRITTEN.

FIRST PART.

BY DANIEL W. FISH, A.M.,

EDITOR OF ROBINSON'S SERIES OF PROGRESSIVE ARITHMETICS.

IVISON, BLAKEMAN, TAYLOR & CO.,
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PREFACE

THE design of the author, in the preparation of the COMPLETE ARITHMETIC, has been to furnish a text-book on the subject of arithmetic, complete not only as a treatise, but as a comprehensive manual for the class-room, and, therefore, embodying every necessary form of illustration and exercise, both *oral* and *written.* Usually, this subject has been treated in such a way as to form the contents of *three or more graded text-books,* the oral exercises being placed in a separate volume. In the present treatise, however, the *whole subject* is presented in all its different grades; and the oral, or mental, arithmetic, so called, has been inserted, where it logically and properly belongs, either as introductory to the enunciation of principles or to the statement of practical rules—the treatment of every topic from the beginning to the end of the book being thoroughly *inductive.*

In this way, and by carefully constructed *analyses,* applied to all the various processes of mental arithmetic, the pupil's mind cannot fail to become thoroughly imbued with clear and accurate ideas in respect to each particular topic before he is required to learn, or apply to written examples, any set rule whatever. The intellect of the pupil is thus addressed at every step; and every part of the instruction is made the means of effecting that *mental development* which constitutes the highest aim, as well as the most important result, of every branch of education.

This mode of treatment has not only the advantage of logically training the pupil's mind, and cultivating his powers of calculation, but must also prove a *source* of *economy,* both of *time* and *money,* inasmuch as it is the means of substituting a *single volume* for an entire *series of text-books.*

As the time of many pupils will not permit them to pursue this study through all of its departments, the work is issued in *two parts,* as well as in a *single volume.* This will, it is thought, be also convenient for *graded schools,* in supplying a separate book for classes of the *higher* and *lower grades* respectively, without requiring any unnecessary repetition or review.

This, the FIRST PART, contains not only all that is essential to a thorough knowledge of the fundamental principles of the subject, but also a great variety of practical exercises, and much valuable information, especially in connection with the article on *Measurements.*

The SECOND PART, which follows this, is a continuation of the subject as taught in the COMPLETE ARITHMETIC, and takes it up where it is left off in this volume, without requiring further study, on the part of the pupil, of any of the elementary topics.

The author feels confident that, on examination, the work as a *whole,* as well as in its separate *parts,* will commend itself to teachers and others, by the careful *grading of its topics,* the clearness and conciseness of its *definitions* and *rules,* its improved methods of *analysis* and *operation,* the use of *equations,* as a form of expression, for arithmetical *drill* and *review,* and the great *number* and *variety* of its examples, both *oral* and *written,* embodying and elucidating all the ordinary business transactions.

Carefully constructed *Synopses for Review* have also been inserted, with the view to afford to both teacher and pupil a ready means of *drill* and *examination,* as well as to present, in a clear, concise, and logical manner, the relations of all the different departments of the subject, with their respective *sub-topics, definitions, principles,* and *rules.*

The author takes pleasure in acknowledging his indebtedness for many valuable suggestions received from teachers of experience, and others interested in the work of education, but especially to Henry Kiddle, A. M., Superintendent of Schools in the city of New York, for important suggestions in relation to various parts of this work.

D. W. F.

BROOKLYN, *January,* 1875.

SUGGESTIONS TO TEACHERS

IN order to teach any subject with the best success, the instructor should not only fully understand it, in all its principles and details, but should also clearly perceive what particular faculties of the mind are concerned in its acquisition and use.

Arithmetic is pre-eminently a subject of practical value; that is, it is one to be constantly applied to the practical affairs of life. But this is true only in a limited sense. Very few ever need to apply to any of the purposes of business more than a small part of the principles and rules of calculation taught in the text-books. Every branch of business has its own requirements in this respect, and these are all confined within very narrow limits.

The teaching of arithmetic must, therefore, to a great extent, be considered as *disciplinary*,—as training and developing certain faculties of the mind, and thus enabling it to perform its functions with accuracy and dispatch. The following *suggestions*, having reference to this two-fold object of arithmetical instruction are presented to the teacher, as a partial guide, not only in the use of this text-book, but in the treatment of the subject as a branch of education.

Seek to cultivate in the pupil the habit of *self-reliance*. Avoid doing for him anything which, either with or without assistance, he should be able to do for himself. Encourage and stimulate his exertions, but do not supersede them.

Never permit him to accept any statement as true which he does not understand. Let him learn not by authority but by demonstration addressed to his own intelligence. Encourage him to ask questions and to interpose objections. Thus he will acquire that most important of all mental habits, that of *thinking for himself*.

Carefully discriminate, in the instruction and exercises, as to which faculty is addressed,—whether that of *analysis* or *reasoning*, or that of *calculation*. Each of these requires peculiar culture, and each has its appropriate period of development. In the *first stage* of arithmetical instruction, *calculation* should be chiefly addressed, and analysis or reasoning employed only after some progress has been made, and then very slowly and progressively. A young child will perform many operations in calculation which are far beyond its powers of analysis to explain thoroughly.

In the exercise of the calculating faculty, the examples should be rapidly performed, without pause for explanation or analysis ; and they should have very great variety, and be carefully arranged so as to advance from the simple and rudimental to the complicated and difficult.

In the exercise of the analytic faculty, great care should be taken that the processes do not degenerate into the mere repetition of *formulæ*. These forms of expression should be as simple and concise as possible, and should be, as far as practicable, expressed in the pupil's own language. Certain necessary points being attended to, the precise form of expression is of no more consequence than any particular letters or diagrams in the demonstration of geometrical theorems. Of course, the teacher should carefully criticise the logic or reasoning, not so as to discourage, yet still insisting upon *perfect accuracy* from the first.

The oral or mental arithmetic should go hand in hand with the written. The pupil should be made to perceive that, except for the difficulty in retaining long processes in the mind, all arithmetic ought to be oral, and that the slate is only to be called into requisition to aid the mind in retaining intermediate processes and results. The arrangement of this text-book is particularly favorable for this purpose.

Definitions and principles should be carefully committed to memory. No slovenliness in this respect should be permitted. A definition is a basis for thought and reasoning, and every word which it contains is necessary to its integrity. A child should not be expected to frame a good definition. Of course, the pupil should be required to examine and criticise the definitions given, since this will conduce to a better understanding of their full meaning.

In conducting recitations, the teacher should use every means that will tend to *awaken thought.* Hence, there should be great variety in the examples, both as to their construction and phraseology, so as to prevent all mechanical ciphering according to fixed methods and rules.

The *Rules* and *Formulæ* given in this book are to be regarded as *summaries* to enable the pupil to retain processes previously analyzed and demonstrated. They need not be committed to memory, since the pupil will have acquired a sufficient knowledge of the principles involved to be able, at any time, to construct rules, if he has properly learned what precedes them.

In the higher department of arithmetic, the chief difficulty consists in giving the pupil a clear idea of the *nature* of the business transactions involved. The teacher should, therefore, strive by careful elucidation, to impart clear ideas of these transactions before requiring any arithmetical examples involving them to be performed. When the exact nature of the transaction is understood, the pupil's knowledge of abstract arithmetic will often be sufficient to enable him to solve the problem without any special rule.

The teacher should be careful not to advance too rapidly. The mind needs time to grasp and hold firmly every new case, and then additional time to bring its new acquisition into relation with those preceding it. Hence the need of frequent *reviews,* in order to give the pupil a comprehensive as well as an accurate and permanent knowledge of this subject.

The *Synopses for Review* interspersed throughout this work are designed for this purpose. The whole or a part of a Synopsis, embracing one or more topics, may be placed upon the blackboard, and the pupil required to give briefly but accurately the *subdivisions, definitions, principles,* etc., involved in each. By this means, if further tested by questions, a thorough and well classified knowledge of the whole subject will be permanently impressed upon his mind.

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COMPLETE ARITHMETIC

FIRST PART.

PRELIMINARY DEFINITIONS

ARTICLE 1. Arithmetic is the Science of Numbers, and the Art of Computation.

As a science, Arithmetic treats of the nature and properties of numbers. As an art, it teaches how to apply a knowledge of numbers to practical and business purposes.

2. A ***Unit*** is one, or a single thing; as *one, one* boy, *one* year, *one* dozen.

3. A ***Number*** is a unit, or a collection of units; as *one, three, five* boys, *six* miles.

4. An ***Integral Number,*** or ***Integer,*** is a number which expresses whole things; as *seven, four* days.

5. The ***Unit of a Number*** is *one* of the collection of units which constitute the number. Thus, the unit of twelve is *one,* of twenty dollars is *one* dollar.

6. A ***Concrete Number*** is a number that is applied to a particular kind of object, or quantity; as three *houses,* four *dollars,* five *minutes.*

7. An ***Abstract Number*** is a number that is not applied to any object; as *four, seven, eight.*

8. ***Like Numbers*** are such as have the same kind of unit, or express the same kind of quantity. They may be either concrete or abstract; as *eight* and *nine,* six *days* and ten *days,* two *rods* five *feet* and five *rods* three *feet.*

9. ***Unlike Numbers*** are such as have different kinds of units, or express different kinds of quantity; as ten *months* and eight *miles;* seven *dollars* and five *barrels.*

10. A ***Scale*** in Arithmetic is a succession of units, increasing and decreasing according to a certain law, or rule. Scales are *uniform* or *varying.*

11. A ***Uniform Scale*** is one in which the law of increase and decrease is the same throughout the entire succession of units.

12. A ***Varying Scale*** is one in which the law of increase and decrease is not the same throughout the entire succession of units.

13. A ***Decimal Scale*** is one in which the law of increase and decrease is *uniformly* TEN.

EXERCISES.

14. 1. How many units in two? In five cents? In six dollars? In seven acres?

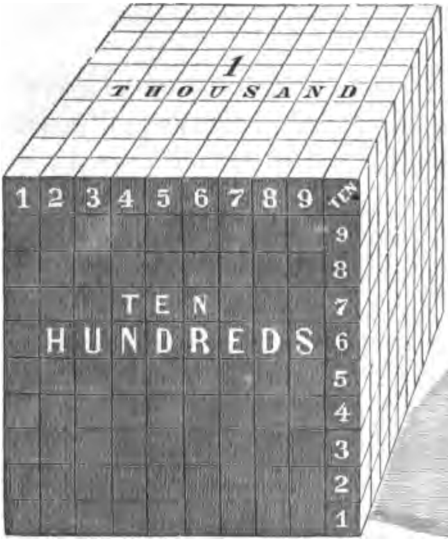
2. Are two trees and five trees like or unlike numbers?

3. Are they concrete or abstract? Why?

4. What kind of numbers are seven and nine? Are five acres and seven cords? Are four coats and six coats?

5. Name two numbers that are *like* and *abstract.*

6. Name two numbers that are *like* and concrete.



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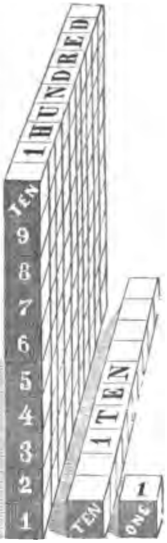


One Hundred and Eleven.

NOTATION AND NUMERATION.

15. In representing numbers, objects are regarded as arranged in *groups* of *tens ;* hence we have single things, or *units ;* next, groups containing *ten* units, or *ten ;* next, groups containing *ten* tens, or *one hundred* ; and again, groups containing *ten* hundreds, or *one thousand,* etc.

16. This method of grouping is called the *Decimal System,* from the Latin word *decem,* which signifies *ten.*

17. ***Notation*** is a method of *writing,* or representing numbers by characters.

18. ***Numeration*** is a method of *reading* numbers *represented by* characters.

19. The number of objects may be represented by *words,* or by *characters.*

20. The characters may be either *figures* or *letters.*

21. ***Figures*** are characters used to express numbers.

22. The ***Arabic Notation*** is the method of expressing numbers by *figures.* It was so called because it was invented by the Arabs.

23. This method employs *ten* different characters, or figures, to represent numbers, viz.:

Figures. 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
Names. One, Two, Three, Four, Five, Six, Seven, Eight, Nine, Naught.

24. The first nine are called *significant* figures, because each has a value of its own. They are also called *digits.*

The *cipher,* or last character, is called *Naught,* or *Zero,* and when standing alone has no value.

These ten characters, when combined according to certain principles, can be made to express any number.

25. The *first nine* numbers are each represented by a *single figure,* and are called *units* of the *first order.*

26. By grouping ten ones, or *units* of the *first* order into a larger collection, there is formed a unit of the *second* order, called *ten,* which is represented by writing the figure 1 with a cipher after it; thus, 10.

27. In the same manner are represented,

Two tens, or Twenty, by	20	Six tens, or Sixty, by	60
Three tens, or Thirty, "	30	Seven tens, or Seventy, "	70
Four tens, or Forty, "	40	Eight tens, or Eighty, "	80
Five tens, or Fifty, "	50	Nine tens, or Ninety, "	90

28. The numbers between *ten* and *twenty* are represented by writing 1 in the *second* place, and the units in the *first* place. Thus,

Eleven	11	Fourteen	14	Seventeen	17
Twelve	12	Fifteen	15	Eighteen	18
Thirteen	13	Sixteen	16	Nineteen	19

29. In like manner, the numbers between 20 and 30 are represented, thus,

Twenty-one	21	Twenty-four	24	Twenty-seven	27
Twenty-two	22	Twenty-five	25	Twenty-eight	28
Twenty-three	23	Twenty-six	26	Twenty-nine	29

30. The greatest number that can be expressed by *two* figures is 99.

31. By grouping ten units of the *second order*, or *ten tens,* into a larger collection, there is formed a unit of the *third* order, called a *hundred,* represented by writing the figure 1 with two ciphers after it; thus, 100.

32. In like manner are represented,

Two hundred	by	200	Six hundred	by	600
Three hundred	"	300	Seven hundred	"	700
Four hundred	"	400	Eight hundred	"	800
Five hundred	"	500	Nine hundred	"	900

33. The numbers from one hundred, to nine hundred and ninety-nine, are represented by writing the *hundreds* in the *third* place, the *tens* in the *second* place, and the *units* in the *first* place.

34. The greatest number that can be expressed by *three* figures is 999.

35. ***Orders of Units*** are denoted by the *position* of the figures used in expressing a number.

Thus, 532 represents 2 units of the *first order*, 3 units of the *second order*, or 3 *tens*, and 5 units of the *third order*, or 5 *hundreds*, and is read five hundred and thirty-two.

36. PRINCIPLES.—1. *Ten units of any order in a number make one unit of the next higher order.*

2. *When any order of units in a number is vacant, the place is filled with a cipher.*

EXERCISES.

37. Express the following numbers by figures :

1. One hundred and twenty.
2. Four hundred and eighty.
3. Seven hundred and six.
4. Five hundred and seven.
5. Seven hundred.
6. Three hundred and eight.
7. Six hundred and ninety.
8. Eight hundred and five.
9. Seven hundred and ten.
10. Six hundred and eleven.
11. Nine hundred and seven.
12. Two hundred and sixty.

38. Copy and read the following, and name the number of *hundreds, tens,* and *units* in each :

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
67	321	190	840	592	219
85	406	761	269	904	807
77	289	345	793	531	395
98	672	402	503	762	608

39. By grouping ten units of the *third* order, or *ten hundreds,* into a larger collection, there is formed a unit of the *fourth* order, called a *thousand,* represented by *writing the figure 1* with three ciphers after it ; thus, 1000.

40. In like manner are represented,

Two thousand	by 2000		Six thousand	by 6000
Three thousand	" 3000		Seven thousand	" 7000
Four thousand	" 4000		Eight thousand	" 8000
Five thousand	" 5000		Nine thousand	" 9000

41. The numbers from one thousand, to nine thousand nine hundred and ninety-nine, are represented by writing *thousands* in the *fourth* place, *hundreds* in the *third* place, *tens* in the *second* place, and *units* in the *first* place.

Thus, 5304 represents 4 units of the *first* order, 0 units of the *second* order, or *tens*, 3 units of the *third* order, or *hundreds*, and 5 units of the *fourth* order, or *thousands*, and is read five thousand three hundred and four.

42. The greatest number that can be expressed by four figures is 9999.

43. In the same manner, other *new* orders are formed to represent larger numbers, by grouping *ten* units of the *fourth* order to form the *fifth* order, or *tens* of thousands; and *ten* units of the *fifth* order, to form the *sixth* order, or *hundreds* of thousands, etc.

Thus, 432076 represents 6 units of the *first* order, 7 units of the *second* order, 0 units of the *third* order, 2 units of the *fourth* order, 3 units of the *fifth* order, and 4 units of the *sixth* order, and is read four hundred thirty-two thousand and seventy-six.

From the preceding illustrations it is obvious, that,

44. Moving a figure one place to the *left*, *increases* its *representative* value *tenfold;* and,

45. Moving a figure one place to the *right*, *diminishes* its *representative* value *tenfold.*

EXERCISES.

46. Write in figures and read :

1. Two units of the third order, four units of the second order, and three units of the first order.

2. Five units of the fourth order, six units of the third order, and two units of the second order.

3. Seven units of the fourth order, eight of the second order, and three of the first.

4. One unit of the third order and four of the second.

5. Three units of the fifth order, two of the third, and one of the first.

6. Eight units of the fourth order, and five of the second.

7. Two units of the sixth order, nine of the fifth, four of the third, one of the second, and seven of the first.

47. Express the following numbers by figures :

1. Thirty-seven thousand.
2. Sixteen thousand one hundred.
3. Twelve thousand five hundred and fifty.
4. Forty-nine thousand five hundred and twenty-seven.
5. Fifteen thousand two hundred and six.
6. Seventeen thousand and twenty-four.
7. Sixty-thousand six hundred and eight.
8. Seven hundred and twenty thousand.
9. Two hundred forty thousand five hundred.

48. Copy, and read the following, naming the number of units of *each order :*

(1.)	(2.)	(3.)	(4.)	(5.)
1542	1020	32507	76387	528031
3473	*1256*	53106	627324	600320

49. This method of numeration groups the successive orders into ***periods*** of *three* figures each. The periods are commonly separated by commas, each period taking the name of its lowest order, as shown in the following

NUMERATION TABLE.

PERIODS.	6th.			5th.			4th.			3d.			2d.			1st.		
NAME.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
ORDERS OF UNITS IN THE PERIODS.	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
NUMBER.		3	0,	2	9	1,	0	4	0,	0	2	7,	3	0	6,	4	0	0

The number is read 30 *quadrillion*, 291 *trillion*, 40 *billion*, 27 *million*, 306 *thousand*, 400.

1. **In *reading* numbers, the name of the units period is omitted.**
2. **Every period except the highest must contain *three* figures.**

50. The names of the periods above Quadrillions are:

PERIODS.	NAMES.	PERIODS.	NAMES.
7th	Quintillions.	15th	Tredecillions.
8th	Sextillions.	16th	Quatuordecillions.
9th	Septillions.	17th	Quindecillions.
10th	Octillions.	18th	Sexdecillions.
11th	Nonillions.	19th	Septendecillions.
12th	Decillions.	20th	Octodecillions.
13th	Undecillions.	21st	Novendecillions.
14th	Duodecillions.	22d	Vigintillions.

51. The pupil may be required to prepare and arrange on the slate or on paper, exercises similar to the following.

The first *example* is read, 341.

The second, is read, 125 *thousand* and 4.

The third, is read, 4 *million* 44 *thousand* and 34.

The fourth, is read, 33 *million* 300 *thousand* 330.

Bill.			Mill.			Thou.			Units.		
H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.
									3	4	1
						1	2	5	0	0	4
					4	0	4	4	0	3	4
				3	3	3	0	0	3	3	0
			2	0	0	2	2	0	0	2	2
		5	0	0	5	5	0	0	0	5	5

The diagram may be prepared at first for only two or three periods, and may be gradually enlarged to five or six periods.

Each pupil may be allowed to dictate an example, to be written and read by the whole class.

52. Rule for Notation.—*Begin at the left, and write the hundreds, tens, and units of each period in their proper order, filling all vacant places and periods with ciphers.*

53. Rule for Numeration.—I. *Begin at the right, and separate the number into periods of three figures each.*

II. *Begin at the left, and read each period as if it were units, giving its name.*

EXERCISES IN NOTATION AND NUMERATION.

54. Write in figures and read:

1. Six units of the 3d order, five of the 2d, and four of the 1st.

2. Five units of the 4th order, seven of the 2d, and six of the 1st.

3. Eight units of the 4th order and four of the 2d.

4. Five units of the 4th order and eight of the 2d.

5. Three units of the 5th order, six of the 4th, four of the 3d, and seven of the 1st.

6. Two units of the 6th order, four of the 5th, nine of the 4th, three of the 3d, and five of the 1st.

7. Three units of the 9th order, eight of the 7th, four of the 6th, six of the 5th, and nine of the 1st.

55. Write and read the following numbers in figures:

1. Twenty-five units in the 2d period, and four hundred ninety-six in the 1st. *Ans.* 25,496.

2. Four hundred thirty-six units in the 4th period, twelve in the 3d, one hundred in the 2d, and three hundred and one in the 1st.

3. Eighty-one units in the 5th period, two hundred and nineteen in the 4th, and fifty-six in the 2d.

4. Nine hundred and forty units in the seventh period, eighteen in the fifth, and one hundred and three in the 3d.

56. Express the following numbers by figures:

1. Twenty-six thousand and twenty-six.
2. Fourteen thousand two hundred and eighty.
3. One hundred seventy-six thousand.
4. Four hundred fifty thousand and thirty-nine.
5. Seven million and thirty-six.
6. Five hundred sixty-three thousand and four.
7. One million ninety-six thousand.
8. Ten million ten thousand ten hundred and ten.
9. Four hundred eighty-three million eight hundred sixteen thousand one hundred and forty-nine.
10. Ninety-nine billion thirty-seven thousand and four.

57. Point off and read the following numbers:

1. 24835.
2. 2474783.
3. 31628045.
4. 247843112.
5. 100103.
6. 53000008.
7. 406270035.
8. 3730016000.

58. SYNOPSIS FOR REVIEW.

PRELIMINARY DEFINITIONS.

- 1. Arithmetic. 2. A Unit. 3. A Number. 4. An Integral Number. 5. Unit of a Number. 6. A Concrete Number. 7. An Abstract Number. 8. Like Numbers. 9. Unlike Numbers. 10. A Scale. 11. A Uniform Scale. 12. A Varying Scale. 13. A Decimal Scale.

NOTATION AND NUMERATION.

- 1. DEFINITIONS.
    - 1. Decimal System. 2. Notation. 3. Numeration. 4. Figures. 5. Arabic Notation.
- 2. MODE OF REPRESENTING
    - 1. Units of the first order.
    - 2. " " second "
    - 3. " " third "
    - 4. " " fourth "
    - 5. Numbers between ten and twenty; between twenty and thirty.
    - 6. Other orders and numbers.
- 3. ORDER OF UNITS—how denoted.
- 4. PRINCIPLES, 1 and 2.
- 5. VALUE OF FIGURES
    - 1. How increased.
    - 2. How diminished.
- 6. PERIODS.
    - 1. Of how many figures.
    - 2. How separated.
    - 3. Names of Periods to Quadrillions.
    - 4. " " " beyond "
- 7. RULES.
    - 1. For Notation.
    - 2. For Numeration.

ADDITION

ORAL EXERCISES.

59. 1. A man gave 5 dollars for a hat, and 9 dollars for a vest. How many dollars did he pay for both?

2. How many miles are 4 miles and 12 miles?

3. How many are 6 men and 14 men? 15 and 7?

4. How many are 14 and 5? 29 and 5? 24 and 5?

5. How many are 15 and 6? 21 and 6? 27 and 6?

6. How many are 7 and 12? 7 and 19? 7 and 26?

7. What kind of numbers are 9 pounds and 14 pounds?

8. Can 9 balls and 12 books be added? Why not?

9. Can 7 rods and 10 rods be added? Why?

10. How many are 4, 5, and 7? 5, 7, and 4? 4, 7, and 5?

11. In a shop are 15 men, 8 boys, and 6 girls at work. How many persons are at work in the shop?

12. I gave 7 cents to one boy, 9 to another, and 6 to another. How many cents did I give to all?

13. Add by 2's from 1 to 31.

OPERATION.—1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31.

Add

14. By 2's from 4 to 50.
15. By 3's from 1 to 43.
16. By 3's from 6 to 51.
17. By 4's from 1 to 53.
18. By 4's from 5 to 45.
19. By 5's from 1 to 61.
20. By 5's from 7 to 82.
21. By 6's from 0 to 72.
22. By 6's from 2 to 80.
23. By 6's from 10 to 88.

24. Add by 2's and 3's alternately from 1 to 31.

OPERATION.—1, 3, 6, 8, 11, 13, 16, 18, 21, 23, 26, 28, 31.

25. Add by 2's and 4's alternately from 1 to 49.

26. Add by 3's and 4's alternately from 0 to 56.

27. Add by 2's and 5's alternately from 4 to 60.

28. Add by 3's and 5's alternately from 7 to 63.

DEFINITIONS AND PRINCIPLES.

60. ***Addition*** is the process of uniting two or more *like* numbers into *one* equivalent number.

61. The ***Sum,*** or ***Amount,*** is the number obtained by addition.

62. The ***Sign of Addition*** is +. It is read *plus,* and signifies *more;* thus, 5 + 6 is read, 5 *plus* 6, and means that 5 and 6 are to be *added.*

63. The ***Sign of Equality*** is =. It is read *equals,* or *equal to;* thus, 5 + 6 = 11, is read 5 *plus* 6 *equals* 11. It may be read 5 *and* 6 *are* 11.

64. An ***Equation*** is an expression of *equality* between two numbers or sets of numbers.

EQUATION.

All that is written *before* the sign of *equality* is called the *first member* of the equation, all that is written *after* the sign of equality is called the *second member.*

The numbers in each member are called the *terms* of the equation.

Thus, $6+4=10$ is an *equation*, and is read 6 *plus* 4 *equals* 10, and means that the sum of 6 and 4 is equal to 10. $6+4$ is the *first member* of the equation, and 10 is the *second member;* and 6, 4, and 10 are the *terms* of the equation.

65. Name the *members* and the *terms* of each of the following equations.

1. $9+12=21$
2. $14+10=24$
3. $26+9=35$
4. $44+12=56$
5. $14+40=44+10$
6. $18+20=31+7$

66. PRINCIPLE.—*Only like numbers and units of the same order can be added.*

EXERCISES.

67. The teacher should read the *first* member of the equation, and the pupil be required, as promptly as possible, to give the *second* member.

The expression "$=?$" is read *equals how many,* or *what.*

$6+4+5=?$
$7+0+2=?$
$8+3+1=?$
$9+2+4=?$
$7+6+5=?$
$10+8+3=?$

$3+12+6=?$
$6+4+10=?$
$2+11+5=?$
$8+9+3=?$
$3+12+7=?$
$14+7+6=?$

$12+5+10=?$
$15+2+4=?$
$13+7+8=?$
$16+5+6=?$
$18+0+10=?$
$20+6+8=?$

$5+8+9=?$
$9+8+5=?$
$8+5+9=?$
$10+7+6=?$
$6+10+7=?$
$11+5+8=?$
$15+3+10=?$

$20+7+3=?$
$8+12+10=?$
$11+9+1=?$
$12+7+9=?$
$21+10+7=?$
$24+6+5=?$
$10+25+9=?$

$4+15+10=?$
$8+8+8=?$
$9+9+9=?$
$23+10+6=?$
$7+21+11=?$
$14+6+12=?$
$26+10+9=?$

WRITTEN EXERCISES.

68. When the sum of the units of each order is less than 10.

1. What is the sum of 421, 44, 303, and 230?

OPERATION.

421
 44
303
230

998 Sum.

ANALYSIS.—Arrange the numbers so that the units of the same order stand in the same column.

Begin with the lowest order of units, and add each column separately; and instead of saying, 3 units and 4 units are 7 units, and 1 unit are 8 units, pronounce the successive results *only;* thus, 3, 7, 8, the sum of the *units,* which write in the units' place.

Next, 3, 7, 9, the sum of the *tens,* which write in the tens' place. Lastly, 2, 5, 9, the sum of the *hundreds,* which write in the hundreds' place. Hence the *sum* is 998.

PROOF.—Add the columns in the reversed direction. If the two results agree, the work is probably correct.

Copy, add, and prove,

(2.)	(3.)	(4.)	(5.)
204	312	241	403
462	243	520	2052
23	124	27	4324

6. I paid 3104 dollars for a house, 450 dollars for repairs, and 234 dollars for painting. What was the whole cost?

69. The ***Sign of Dollars*** is $. It is read, *dollars.* Thus, $35 is read 35 *dollars;* $9 is read 9 *dollars.*

70. When *dollars* and *cents* are written, a period or point (.) is placed before the cents, or between the dollars and cents. Thus, $7.25 is read 7 dollars and 25 cents.

71. Since 100 cents make $1.00, *cents* always occupy *two* places, and never more than two.

72. If the number of cents is less than 10 and expressed by a single figure, a *cipher* must occupy the first place at the right of the point. Thus, 3 dollars 6 cents are written \$3.06; 1 dollar 5 cents are written \$1.05.

73. When cents alone are written, and their number is less than 100, either write the *word* cents *after* the number, or place the dollar sign and the point *before* the number. Thus, 75 cents may be expressed, \$.75.

74. In arranging for addition, dollars should be written under dollars, and cents under cents, in such order that the *points* stand in a *vertical* line.

The *sign* \$, and the *point* (.) should never be omitted.

75. Read the following equations:

1. $\$12. + \$8. = \$20.$
2. $\$25. + \$10. = \$35.$
3. $\$3.25 + \$6.75 = \$10.$
4. $\$.75 + \$.20 = \$.95.$
5. $\$.60 + \$.40 = \$1.00.$
6. $\$14.08 + \$3.14 = \$17.22.$

76. Express the following by proper figures and signs:

1. Nine dollars and thirty cents.
2. Thirty dollars and ten cents.
3. Eighty-four cents.
4. Seventy-eight cents.
5. Six dollars and sixteen cents.
6. 7 dollars and 26 cents.
7. 9 dollars and 5 cents.
8. 19 dollars and 7 cents.
9. 69 cents; 23 cents.
10. 10 cents; 6 cents.

The teacher may exercise the class *orally*, by dictating rapidly, but distinctly, similar examples. Thus, *Sign, five, three?* The prompt response should be, "Fifty-three dollars" (\$53). *Ques. Sign, point, seven, four? Ans.* Seventy-four cents (\$.74). *Ques. Sign, point, naught, eight? Ans.* Eight cents (\$.08), etc.

Also, the converse; thus, *Ques.* "Forty-five dollars" (\$45)? *Ans. Sign, four, five. Ques.* Fifty-six cents (\$.56)? *Ans. Sign, point, five, six. Ques.* Nine dollars seven cents (\$9.07)? *Ans. Sign, nine, point, naught, seven*; etc.

77. Copy and add,

(1.)	(2.)	(3.)	(4.)
$3.04	$24.12	$105.	$200.35
2.21	3.06	32.14	46.41
.53	12.	.73	1.02

5. What is the sum of $.25, $3.31, $14.02, and $21.

6. What is the sum of ten dollars and twenty cents, four dollars and fifteen cents, forty-three cents, and thirteen dollars ?

7. Bought a horse for $154, and sold him for $35.75 more than he cost. For how much did I sell him ?

8. A lady paid 12 dollars for a scarf, 3 dollars and 25 cents for a fan, two dollars for a pair of gloves, and 42 cents for a collar. How much did she pay for all ?

It is not the design to teach here, the *principles* and *reductions* of Decimal Currency, fully taught in another place, but to give a few hints and illustrations, to enable the teacher, by oral instruction and simple written exercises, to make the pupil familiar with the *use* of decimal currency in the common business matters of life ?

ORAL EXERCISES.

78. 1. How many are 7 and 6 ? 13 and 6 ? 19 and 6 ? 25 and 6 ? 31 and 6 ? 42 and 6 ?

2. How many are 9 and 7 ? 16 and 7 ? 23 and 7 ?

Add

3. By 7's from 1 to 71.
4. By 7's from 3 to 87.
5. By 8's from 0 to 96.
6. By 8's from 6 to 102.
7. By 9's from 2 to 92.
8. By 9's from 10 to 109.
9. By 10's from 0 to 120.
10. By 10's from 13 to 153.
11. By 11's from 1 to 100.
12. By 11's from 4 to 92.
13. By 12's from 0 to 144.
14. By 12's from 3 to 135.

Add rapidly the following:

15. 4, 6, 5, 3, and 7.
16. 6, 4, 8, 2, and 5.
17. 10, 9, 5, 3, and 6.
18. 7, 3, 10, 9, and 8.
19. 14, 5, 3, 6, and 10.
20. 13, 5, 6, 10, and 3.
21. 12, 10, 2, 0, and 9.
22. 27, 3, 10, 8, and 7.
23. 36, 12, 7, 4, and 10.
24. 11, 12, 10, 9, and 8.

Add by repeating the numbers,

25. 2, 3, 4, 2, 3, 4, 2, 3, 4, till the sum = 63.
26. 3, 4, 5, 3, 4, 5, 3, 4, 5, till the sum = 84.
27. 2, 4, 6, 2, 4, 6, 2, 4, 6, till the sum = 96.

Add alternately,

28. 5, 6, 5, 6, 5, 6, 5, 6, till the sum = 88.
29. 6, 4, 6, 4, 6, 4, 6, 4, 6, 4, till the sum = 100.
30. 7, 5, 7, 5, 7, 5, 7, 5, 7, 5, till the sum = 120.
31. 8, 9, 8, 9, 8, 9, 8, 9, 8, 9, till the sum = 119.

32. What is the sum of 46 and 27?

ANALYSIS.—46 is 4 tens and 6 units, and 27 is 2 tens and 7 units; 4 tens and 2 tens, are 6 tens, and 6 units and 7 units are 13 units, or 1 ten and 3 units, which added to 6 tens make 7 tens and 3 units, or 73.

33. $36+42=?$
34. $53+38=?$
35. $65+40=?$
36. $54+38=?$
37. $29+61=?$
38. $38+37=?$
39. $44+37=?$
40. $72+25=?$
41. $63+54=?$

42. James earned 44 cents one day, and 52 cents the next. How many cents did he earn in both days?

43. Bought a pound of coffee for 35 cents, a pound of butter for 28 cents, and a pound of sugar for 12 cents. What was the cost of the whole?

44. A lady bought a silk dress for $28, a shawl for $16, and had $14 left. How much money had she at first?

WRITTEN EXERCISES.

79. When the sum of the units of any order equals or exceeds 10.

1. What is the sum of 467, 536, 84, and 705?

OPERATION.
467
536
84
705
1792 Sum.

ANALYSIS.—Arranging the numbers as before, begin at the right hand and add the column of units; thus, 5, 9, 15, 22 *units*, equal to 2 tens and 2 units. Write the 2 units in the units' place, and reserve the 2 tens to add to the next column.

Next, adding the 2 tens reserved, to the column of tens, say, 2, 10, 13, 19 *tens*, equal to 1 hundred and 9 tens. Write the 9 tens in the tens' place, and reserve the 1 hundred to add to the next column.

Lastly, adding the 1 hundred reserved, say 1, 8, 13, 17 *hundreds*, equal to 1 thousand, and 7 hundreds, which write in hundreds' and thousands' places. Hence the *sum* is 1792.

In like manner, copy, add, and prove,

(2.)	(3.)	(4.)	(5.)
276 miles.	876 feet.	$20.30	$145.24
307 "	94 "	7.56	36.60
638 "	142 "	13.08	105.08
425 "	507 "	25.	.75

6. Find the sum of $370.21, $2.49, $3.07, and $.94.

7. Find the sum of 2008, 1400, 706, 300, and 77.

8. If 4 loads of coal weigh respectively 1922, 1609, 2100, and 1873 pounds, what is the entire weight?

RULE.—I. *Write the numbers so that figures of the same order stand in the same column.*

II. *Beginning at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

III. *If the sum of any column consists of two or more figures, write the unit figure under that column, and add the remaining figure or figures to the next column.*

PROOF.—*Add each column in the reverse direction. If the results agree, the work is correct.*

9. The Duke of Wellington's army at Waterloo consisted of 26661 infantry, 8735 cavalry, 6877 artillery, and 33413 allies. What was the whole number of his army?

10. Napoleon's army at Waterloo was composed of infantry 48950, cavalry 15765, and artillery 7732. What was the whole number of his army?

11. Gave $325 for a horse, $275.50 for a carriage, $75.75 for a harness, and $20.62 for a robe. What was the cost of the whole?

12. Bought a pair of boots for $8.50, an umbrella for $3.62, a pair of gloves for $1.25, some collars for $.75, and a hat for $4. What was the whole cost?

13. A lady gave $48.50 for silk for a dress, $16.75 for the trimmings, and $15.62 for making. What was the cost of the dress?

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
$99.84	96256	$117.76	98304	1728	$675.84
24.96	6016	29.44	6144	864	168.86
6.24	376	7.36	384	108	10.56
1.56	141	1.84	24576	81	1.32
12.48	188	3.68	3072	5296	.96
.98	1504	58.88	144	3456	2.64
3.12	752	1.38	49152	432	84.48

20. What is the sum of 5736 dollars and 45 cents, 1000 dollars and 80 cents, 405 dollars and 15 cents, 50 dollars and 9 cents, and 79 cents?

21. Find the sum of twenty-five hundred dollars, 420 dollars and 47 cents, $23 and fifty cents, $600, and ten dollars and eight cents.

22. 1 million 400 thousand and 50 + 15 hundred + 25 thousand + 120 thousand 6 hundred and 14 = ?

23. Paid $3456 for a house, $426.75 for painting it, $2809.48 for furniture. What was the cost of the whole?

24. North America has an area of 8825537 square miles, South America 6954131 square miles, and the West Indies 93810 square miles. What is the area of the entire American Continent?

25. A man owns farms valued at $62500, city lots worth $10260, a house worth $21300, and other property to the amount of $10500. What is the total value of his property?

80. SYNOPSIS FOR REVIEW.

ADDITION.
- 1. DEFINITIONS.
  - 1. Addition. 2. Sum or Amount. 3. Sign of Addition. 4. Sign of Equality. 5. An Equation. 6. Members and Terms of an Equation.
- 2. PRINCIPLES, 1 and 2.
- 3. ADDITION OF DOLLARS AND CENTS.
  - 1. Sign of Dollars.
  - 2. Use of the Period.
  - 3. Number of places for cents.
  - 4. Mode of expressing cents.
  - 5. How to arrange for Addition.
- 4. Rule, I, II, III.
- *5. PROOF.*

SUBTRACTION

ORAL EXERCISES.

81. 1. If John is 15 years old and George is 6, what is the difference in their ages?

2. How many are 16 cents less 7 cents?
3. How many are 18 dollars less 5 dollars?
4. How many are 14 less 6? 16 less 4? 12 less 5?
5. How many are 18 less 8? 20 less 6? 21 less 4?
6. Five balls taken from 11 balls leave how many?
7. Six cents from 20 cents leave how many?
8. What kind of numbers are 10 days and 6 pounds?
9. Can 6 miles be taken from 15 acres? Why not?
10. Can 8 dollars be taken from 18 dollars? Why?
11. How many are 7 less 5? 17 less 5? 27 less 5?
12. How many are 9 less 6? 19 less 6? 29 less 6?
13. What number added to 8 will make 12?
14. What number and 9 make 13? 14? 15? 16?
15. Subtract by 2's from 24 to 0.

Operation.—24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.

In the same manner, subtract

16. By 2's from 25 to 1.
17. By 2's from 31 to 3.
18. By 3's from 30 to 0.
19. By 3's from 37 to 1.
20. By 3's from 40 to 4.
21. By 4's from 44 to 0.
22. By 4's from 41 to 1.
23. By 4's from 51 to 3.
24. By 5's from 60 to 0.
25. By 5's from 63 to 3.
26. By 6's from 66 to 0.
27. By 6's from 65 to 5.

28. Count by 4's from 2 to 58, and back from 58 to 2.
29. Count by 5's from 1 to 61 and back to 1.
30. Count by 6's from 3 to 69 and back to 3.
31. Count by 4's from 5 to 53 and back to 5.
32. Count by 6's from 7 to 67 and back to 7.

DEFINITIONS.

82. ***Subtraction*** is the process of finding the difference between two like numbers.

83. The ***Difference,*** or ***Remainder,*** is the result obtained by subtracting.

84. The ***Minuend*** is the number from which another number is to be subtracted.

85. The ***Subtrahend*** is the number to be subtracted.

86. The ***Sign of Subtraction*** is —. It is read *minus,* and signifies *less.*

When placed between two numbers, it indicates that the one *after* it is to be subtracted from the one *before* it. Thus, 12 — 7 is read 12 *minus* 7, and means that 7 is to be *subtracted* from 12.

87. A ***Parenthesis*** ( ) is used to include within it such numbers as are to be considered together. A ***Vinculum*** $\overline{\qquad\quad}$ has the same signification. Thus, $15 + (12 - 7)$, or $15 + \overline{12 - 7}$, signifies that to 15 is to be added the difference between 12 and 7.

88. PRINCIPLES.—1. *Only like numbers and units of the same order can be subtracted.*

2. *The minuend must be equal to the sum of the subtrahend and remainder.*

EXERCISES.

89. To be given in the same manner as those in Art. **67.**

14−9= ?
15−6= ?
22−8= ?
13−7= ?

24− 9= ?
11− 6= ?
17−12= ?
23− 9= ?

21−12= ?
18−10= ?
22− 8= ?
19− 7= ?

21− 9= ?
26− 7= ?
20−12= ?
25−11= ?

16+ 7−10= ?
20+ 9− 7= ?
23+ 5−12= ?
24+ 6−11= ?

19−12+11= ?
22−11+15= ?
26−10+14= ?
29− 8+ 6= ?

What is the difference between

17 and 4+6 ?
24 and 9+5 ?
18 and 7+7 ?

20 and 6+ 6 ?
35 and 9+20 ?
28 and 9+ 9 ?

12+ 5 and 24+3 ?
27− 8 and 14+8 ?
30−10 and 9+8 ?

Similar dictation exercises may be given by the teacher.

WRITTEN EXERCISES.

90. When the figure of the subtrahend is not greater than the corresponding figure of the minuend.

1. From 798 subtract 563.

OPERATION.

Minuend	7 9 8
Subtrahend	5 6 3 −
Remainder	2 3 5

ANALYSIS.—Write the less number under the greater, so that units of the same order stand in the same column.

Begin at the right, and subtract each order of units separately; thus, 3 units from 8 units leave 5 units, which write in the units' place; 6 tens from 9 tens leave 3 tens, which write in the tens' place; 5 hundreds from 7 hundreds leave 2 hundreds, which write in the hundreds' place. Hence the remainder is 235.

The remainder 235 added to the subtrahend 563 equals 798, the minuend. *Hence the work is correct.* (PRIN. 2.)

Copy, subtract, and prove,

	(2.)	(3.)	(4.)	(5.)
Minuend	426	573	784	837
Subtrahend	214	321	434	315

	(6.)	(7.)	(8.)	(9.)
From	624 feet.	795 tons.	864 men.	$976
Take	211 "	352 "	413 "	$525

91. Before subtracting dollars and cents, the numbers must be written as in Addition, in such order that the points will stand in the same *vertical* line.

In like manner, subtract and prove,

10. $54.26 from $68.37.
11. 2714 from 5945.
12. $30.52 from $81.76.
13. 1763 tons from 3886 tons.
14. 6245 feet from 8569 feet.
15. 7301 days from 9625 days.

16. $93.64— $52.41= ?
17. $270.59— $40.16= ?
18. $703.42—$501.30= ?
19. 437615—213502= ?
20. 732740— 11520= ?
21. 242674— 32142= ?

Find the difference between

22. 1204 and 5379.
23. 1370 and 1421.
24. 8673 and 3560.
25. $57.46 and $18.00+$24.25.
26. $50.20+$4.01 and $76.31.
27. $98.76 and $30.46+$43.04.

28. From five thousand seven hundred and forty, take 3 thousand and 30.

29. From 46 thousand 5 hundred and 27, take 12 thousand 3 hundred and fourteen.

30. Two men bought a piece of property for $358.50. One paid $146.30; how much did the other pay?

31. A house and lot sold for $7856, which was one thousand one hundred and ten dollars more than it cost. What did it cost?

32. A certain city has a population of 246857, which is 25324 more than it contained last year. What was its population last year?

ORAL EXERCISES.

92. 1. A man having $20, paid $7 for a hat, and $8 for a vest. How many dollars had he left?

ANALYSIS.—The difference between $20, and the sum of $7 and $8, which is $5. Hence he had $5 left.

2. A boy had 25 cents, and gave 15 cents for a slate and 10 cents for some paper. How many cents had he left?

3. Ella having 16 cents, Jane gave her 9 more, and James gave her enough to make her number 36. How many did James give her?

4. Subtract by 7's from 63 to 0.
5. By 7's from 80 to 3.
6. By 8's from 64 to 0.
7. By 8's from 85 to 5.
8. By 9's from 90 to 0.
9. By 9's from 86 to 5.
10. By 10's from 100 to 0.
11. By 11's from 119 to 9.
12. By 11's from 125 to 4.
13. By 12's from 129 to 9.
14. By 12's from 150 to 6.
15. Count by 7's from 2 to 86, and back from 86 to 2.
16. Count by 8's from 4 to 100, and back to 4.
17. Count by 9's from 7 to 115, and back to 7.
18. Count by 10's from 16 to 136, and back to 16.
19. Count by 11's from 9 to 119, and back to 9.
20. Count by 12's from 20 to 140, and back to 20.
21. How many are 5 tens less 3 tens? 50 — 30?

22. How many are 6 tens less 4 tens? $60 - 40$?

23. From 6 tens 5 units subtract 4 tens 3 units.

24. From 8 tens 7 units subtract 5 tens 6 units.

25. From a cask containing 52 gallons, 27 gallons were drawn out. How many gallons remained?

ANALYSIS.—The difference between 52 gallons and 27 gallons. 27 is 2 tens and 7 units. 2 tens or 20 from 52 leaves 32, and 7 from 32 leaves 25. Hence 25 gallons remained in the cask.

26. From a piece of cloth containing 46 yards, 24 yards were cut. How many yards were left?

27. A man bought a watch for $40, and a chain for $15, and sold both for $63. How much did he gain?

28. How many are 6 and 40, less 5 and 20?

29. How many are 7 and 30, taken from 5 and 50?

30. Eighteen plus 12 equals 40 minus how many?

31. Twenty-two plus 15, equals how many plus 10?

32. William having 75 cents, gave 25 cents for a book and 20 cents for a slate. How many cents had he left?

33. A farmer sold a horse for $96, which was $23 more than the horse cost. What did he cost?

Find the *omitted* term in the following equations:

34. $12+8-6=?$
35. $46+12-14=?$
36. $57-13+8=?$
37. $60-(24+6)=?$
38. $28+6=40-?$
39. $42-12=18+?$
40. $30+25=?+40$
41. $27-11=19-?$
42. $36+16=60-?$
43. $54-12=?+12$
44. $17+23=56-?$
45. $18+25=23+?$
46. $64-48=30-?$
47. $75-30=?+15$
48. $16+38=60-?$
49. $43+?=27+28$
50. $80-?=100-40$
51. $22+54=64+?$

WRITTEN EXERCISES.

93. When any figure of the subtrahend is greater than the corresponding figure of the minuend.

1. From 953 subtract 674.

OPERATION.

	8 14 13
Minuend	9 5 3
Subtrahend	6 7 4
Remainder	2 7 9

ANALYSIS.—Write the numbers as before (**90**), and subtract each order of units separately.

Since 4 units cannot be subtracted from 3 units, increase the 3 units by a unit from the next higher order, or 10 units, making 13 units. 4 units from 13 units leave 9 units, which write in the units' place.

Since 1 of the tens was united with the units, there are 4 tens left. As 7 tens cannot be subtracted from 4 tens, increase the 4 tens by a unit from the next higher order, or 10 tens, making 14 tens. 7 tens from 14 tens leave 7 tens, which write in the tens' place.

Since 1 of the hundreds was united with the tens, there are 8 hundreds left. 6 hundreds from 8 hundreds leave 2 hundreds, which write in the hundreds' place. Hence the remainder is 279.

In like manner, solve and prove the following:

	(2.)	(3.)	(4.)	(5.)
From	3273	6345	5702	7465
Subtract	1425	2462	4384	3270

	(6.)	(7.)	(8.)
From	42670 miles.	51062 acres.	246700 feet.
Take	14384 "	24300 "	18030 "

When one of the given numbers contains *cents*, and the other does not, fill the vacant places with two ciphers.

	(9.)	(10.)	(11.)	(12.)
From	$325.17	$279.00	$105.08	$7.00
Take	84.36	183.42	67.00	.84

RULE.—I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

II. *Begin at the right, and subtract the units of each order of the subtrahend from the units of the corresponding order of the minuend, and write the result beneath.*

III. *If the units of any order of the subtrahend are greater than the units of the corresponding order of the minuend, increase the latter by* 10, *and subtract; then diminish by* 1 *the units of the next higher order in the minuend, and proceed as before.*

PROOF.—*Add the remainder to the subtrahend, and if the sum is equal to the minuend, the work is correct.*

Instead of *diminishing* by 1 the units of the next higher order in the *minuend*, we may *increase* by 1 the units of the next higher order in the *subtrahend.*

Subtract	From
13. 20762 from 53120.	16. $430.09, take $272.46.
14. $73.16 from $138.	17. 15200 rods, take 6472 rods.
15. $247 from $382.28.	18. 120764 tons, take 75028 tons.

How many years from the date of each of the following events to the present year?

19. Figures were used by the Arabs in the year 890.

20. Decimal fractions were invented in 1464.

21. Printing was invented in 1441.

22. The telescope was invented by Galileo in 1610.

23. The electric telegraph was first used in the United States in 1844.

24. The first passage of the Atlantic Ocean by steam was in 1839.

What is the difference between

25. 34726 and 47062 ?
26. 57600 and 20012 ?
27. 70361 and 1005 ?
28. 7620 and 12420 ?
29. $4027 and $703.41 ?
30. $1076 and $2340.50 ?
31. 2762 + 10341 and 45701 + 1200 ?
32. 3000 + 42301 and 720 + 1684 + 7342 ?

33. A merchant bought a quantity of goods for $1248.65, and sold them for $1540. How much did he gain ?

34. Sold a horse for $250.75, which was $28 more than he cost. How much did he cost ?

35. A man having $15740.80, gave $5085 for a store, and $7640.75 for goods. How much money had he left ?

36. If a piece of property bought for $7086.86 is sold at a loss of $1562.09, for how much is it sold ?

Find the *second member* of the following equations :

37. 12346 + 840 + 1046 − 3846 = ?
38. $210 + $809.76 − ($15.21 + $308.76) = ?
39. $600.09 − $276.25 + ($5682 − $654) = ?
40. $1032.07 + $68.05 + $.98 − $1000 = ?
41. 476281 − 12672 − 8720 + 20000 = ?

REVIEW.

ORAL EXAMPLES.

94. 1. The sum of two numbers is 46, and one of them is 18 ; what is the other ?

2. The difference of two numbers is 16, and the greater is 32 ; what is the less ?

3. The difference of two numbers is 24, and the less is 26 ; what is the greater ?

4. A boy having 28 peaches gave 8 to his brother, 7 to his sister, and ate 4; how many had he left?

5. If a lady buy some thread for 10 cents, some needles for 5 cents, and some ribbon for 20 cents, and give the clerk 50 cents, how much change should he return?

6. In a garden are 47 fruit trees; 15 of them are peach trees, 12 plum trees, and the remainder pear trees. How many pear trees are there?

7. A lady having 3 ten-dollar bills and 1 five-dollar bill, bought a bonnet for \$11, a pair of gaiters for \$7, and a scarf for \$3. How much money had she left?

8. A man died at the age of 64 years, having been married 36 years. What was his age when he married?

9. In a public school there are 75 pupils, and 47 of them are girls; how many of them are boys?

10. A man sold 25 sheep, then bought 12, and then had 20. How many had he at first?

11. A merchant gave \$52 for a box of goods, and paid \$5 freight; for how much must he sell them to gain \$15?

12. A man gave his watch and \$10 in money for a harness valued at \$75. How much did he get for his watch?

13. A man having received \$45 for labor, paid \$15 for a coat, \$7 for a barrel of flour, and \$6 for a ton of coal. How much had he left?

14. A man bought a vest for \$7, a pair of pants for \$12, and three shirts for \$9, and gave in payment 3 ten-dollar bills. How much change should he receive?

Find the *required* term in the following equations:

15. $42-(10+12)=?$
16. $9+16=30-?$
17. $36-14=15+?$
18. $36-(8+9)+12=?$
19. $7+16-8=22-?$
20. $14+\overline{28-16}-9=?$

WRITTEN EXAMPLES.

95. 1. The subtrahend is 260346, and the remainder 72304. What is the minuend?

2. The difference is $310.62, and the minuend $1206.28. What is the subtrahend?

3. What is the sum of 4062 and 12356 increased by the difference between 15000 and 975?

4. From the sum of 23462 and 9030, subtract the difference between 34000 and 7640.

5. From the difference between 19876 and 6032, subtract the difference between 12000 and 673.

6. From what sum must $.62 be taken to leave a remainder of $14.60?

7. There were 67374 miles of railway in the United States in 1872, and 71564 miles in 1873. How much was the gain in one year?

8. A man has $10000. How much must he add to this, to be able to pay for a farm worth $13640?

9. California contains 158933 square miles, and Texas 237321 square miles. How much larger is Texas than California?

10. Mt. Blanc is 15572 feet high, and Pike's Peak 12000 feet. What is the difference in their height?

11. A man willed $125000 to his wife and two children. To his son he gave $44675, to his daughter $26380, and the remainder to his wife. What was his wife's share?

12. A merchant of Nashville goes to New Orleans with $21600. He invests $7638.50 in groceries, $3210.65 in crockery, $1245.18 in woodenware, and the remainder in hardware. How much does he invest in hardware?

13. The population of London in 1870 was 3250000; of New York, 944292; and of Brooklyn, 396099. How much greater was the population of London than of New York and Brooklyn?

14. A man owns property valued at $75860, of which $45640 is invested in real estate, $25175.75 in personal property, and the remainder he has in bank. How much has he in bank?

15. Three persons bought a hotel valued at $42075. The first agreed to pay $8375.50, the second agreed to pay twice as much, and the third the remainder. How much was the third to pay?

16. A had $725.40, B had $180.36 more than A, and C had as much as A and B together minus $214. How much had C?

17. 376+1684+573−(931+1000)= ?
18. $27.62+($30.50−$14.00)−$7.62= ?
19. 17300+6840−(5800+1386)=25300− ?
20. (48036−7690)−(3600+1873)=18321+ ?

96. SYNOPSIS FOR REVIEW.

SUBTRACTION.
- 1. DEFINITIONS.
  - 1. Subtraction. 2. Difference, or Remainder. 3. Minuend. 4. Subtrahend. 5. Sign of Subtraction. 6. A Parenthesis, or Vinculum.
- 2. PRINCIPLES, 1 and 2.
- 3. SUBTRACTION OF DOLLARS AND CENTS.
  - 1. How the numbers should be written.
  - 2. If one number contains cents, and the other does not.
- 4. Rule, I, II, III.
- *5. PROOF.*

MULTIPLICATION

ORAL EXERCISES.

97. 1. If a man earns $3 a day, how many times $3 does he earn in 4 days? $3 + $3 + $3 + $3 are how many?

2. There are 7 days in 1 week. How many days are there in 3 weeks? How many are three 7's, or 3 times 7?

3. There are 4 pecks in 1 bushel. How many pecks in 4 bushels? Four 4's, or 4 times 4 are how many?

4. Is the result the same whether we say 4 times 6, or 6 times 4?

5. What is the difference between six 5's and five 6's?

6. How many are three 8's? Eight 3's?

7. Add by 2's from 0 to 24.

8. Multiply from 0 times 2, to 12 times 2.

OPERATION.—0 times 2 is 0, once 2 is 2, twice 2 are 4, 3 times 2 are 6, 4 times 2 are 8, 5 times 2 are 10, and so on.

9. Subtract by 2's back from 24 to 0.

10. Multiply back from 12 times 2 to 0 times 2.

OPERATION.—12 times 2 are 24, 11 times 2 are 22, 10 times 2 are 20, 9 times 2 are 18, 8 times 2 are 16, and so on.

11. Multiply from 0 times 3 to 12 times 3, and back.

12. Multiply from 0 times 4 to 12 times 4, and reverse.

13. Multiply from 0 times 5 to 12 times 5, and reverse.

14. Multiply from 0 times 6 to 12 times 6, and reverse.

Multiplication Table.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

DEFINITIONS.

98. ***Multiplication*** is the process of taking one of two numbers as many times as there are units in the other. Or, it is a short method of *adding equal* numbers.

99. The ***Multiplicand*** is the number to be multiplied.

100. The ***Multiplier*** is the number by which to multiply. It shows *how many times* the multiplicand is to be taken.

101. The ***Product*** is the result obtained by the multiplication.

The multiplicand and multiplier are called the *factors* of the *product*.

102. The ***Sign of Multiplication*** is ×. It is read *times*, or *multiplied by*

When placed between two numbers, it shows that they are to be multiplied together. Thus, 9 × 7 is read 9 *multiplied by* 7, or 7 *times* 9.

Since changing the order of the factors does not change the result, 9 × 7 may be read, 7 *times* 9, or 9 *times* 7.

103. PRINCIPLES.—1. *The multiplier is always regarded as an abstract number.*

2. *The multiplicand and product are like numbers, and may be either concrete or abstract.*

In examples containing concrete numbers, the concrete number is the *true multiplicand*, but when it is the smaller, it is often, for convenience, used *abstractly* as the multiplier.

ORAL EXERCISES.

104. $8\times4=?$

$7\times7=?$

$6\times9=?$

$0\times5=?$

$10\times8=?$

$10\times0=?$

$9\times5=?$

$7\times11=?$

$12\times6=?$

$9\times7=?$

$11\times8=?$

$9\times12=?$

$11\times10=?$

$10\times12=?$

$9\times11=?$

$8\times8+10=?$

$9\times4-10=?$

$12\times6+15=?$

$10\times12-25=?$

$8\times11-12=?$

$9\times9+19=?$

$10\times10-14=?$

$7\times12+16=?$

$8\times0-7=?$

$0\times12\times8=?$

$1\times12+8=?$

$12\times10-30=?$

$15\times2+15=?$

$11\times11-9=?$

$12\times0+25=?$

$10\times12-16=?$

$12\times11-12=?$

$12\times12+6=?$

1. At 7 cents each, what is the cost of 5 pencils?

ANALYSIS.—Since 1 pencil costs 7 cents, 5 pencils will cost 5 times 7 *cents, or 35 cents.* Hence 5 pencils cost 35 cents.

2. What is the cost of 4 tons of coal, at $8 a ton?

3. What is the cost of 5 hats, at $5 a piece?

4. At 9 cents each, what will 3 melons cost?

5. What will 5 yards of gimp cost, at 11 cents a yard?

6. 12 inches make a foot. How many inches in 4 feet?

7. At $4 a cord, what will 9 cords of wood cost?

8. Multiply from 0 times 6 to 12 times 6, and reverse.

9. Multiply from 0 times 7 to 12 times 7, and reverse.

10. Multiply from 0 times 8 to 12 times 8, and reverse.

11. Multiply from 0 times 9 to 12 times 9, and reverse.

12. What cost 6 pairs of boots at $8 a pair? At $9?

13. At 8 cents each, what cost 9 books? 10 books?

14. What cost 9 barrels of flour at $9 a barrel? At $10?

15. 7 days make a week. How many days in 7 weeks?

16. If a man earn $12 in 1 week, how much will he earn in 8 weeks? In 9 weeks?

17. Multiply from 0 times 10 to 12 times 10, and reverse.

18. Multiply from 0 times 11 to 12 times 11, and reverse.

19. Multiply from 0 times 12 to 12 times 12, and reverse.

20. At 12 cents a yard, what cost 9 yards of calico?

21. What cost 10 pounds of ginger, at 11 cents a pound?

22. At $11 a hundred, what will 11 hundred posts cost?

23. How many bushels of grain can be put in 8 bins, each containing 12 bushels?

24. How many are 8 times $4, minus $7?

25. How many are 7 times 9 pounds, plus 10 pounds?

26. How many are 6 times 12 rods, less 20 rods?

27. James gave 5 cents each for 6 oranges. How much change should he receive for 50 cents?

28. How much more than $35 will 7 tons of coal cost, *at $6 a ton?*

WRITTEN EXERCISES.

105. When the multiplier consists of but one order of units.

1. How many are 4 times 73?

1ST OPERATION.

$$\begin{array}{lr} & 73 \\ & 73 \\ & 73 \\ & \underline{73} \\ \text{Sum} & 292 \end{array}$$

ANALYSIS.—To obtain the result by Addition. First find the sum of four 3's, or 4 *times* 3 units, which is 12 units, equal to 1 ten and 2 units. Write the 2 units in the units' place, and reserve the 1 ten to be added to the sum of the tens.

Next the sum of four 7's, or 4 *times* 7 tens, is 28 tens, plus 1 ten reserved are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' place. Hence the *sum* is 292.

2D OPERATION.

$$\begin{array}{lr} \text{Multiplicand} & 73 \\ \text{Multiplier} & \underline{4} \\ \text{Product} & 292 \end{array}$$

ANALYSIS.—In this operation, the multiplicand 73 is written but once; and as it is to be taken 4 times, write the multiplier 4 under it, and commence at the right to multiply. 4 times 3 units are 12 units, or 1 ten and 2 units. Write the 2 units in units' place and reserve the 1 ten to add to the product of the tens.

Next, 4 times 7 tens are 28 tens, plus 1 ten reserved, are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' places. Hence the *product* is 292, equal to the *sum* in the first operation.

Solve by both methods,

2. 3 times 84.
3. 4 times 135.
4. 5 times 234.
5. 6 times 352.
6. 4 times $204.
7. 5 times $425.
8. Multiply 4621 by 4; by 5; by 6; by 7.
9. Multiply 3062 by 6; by 7; by 8; by 9.

What is the product

10. Of $5642 by 6? by 5? by 7? by 9?
11. Of 20372 feet by 7? by 9? by 5? by 6?

12. What cost 527 barrels of flour, at $9 a barrel?

Although $9 is the *true multiplicand,* for convenience, we may use 9 for the multiplier, and 527 as the multiplicand (**103,** Note), but the *product* is *dollars,* since the true multiplicand is dollars. This is obvious, since 527 barrels at $1 a barrel, would cost $527, and at $9 a barrel, 9 times $527, etc.

13. What cost 326 tons of coal, at $6 a ton?

14. What cost 1238 cords of wood, at $5 a cord?

15. What cost 752 pounds of nails, at 7 cents a pound?

OPERATION.—7 cents × 752 = 5264 cents = $52.64.

When *either* factor contains cents, the *product* is *cents,* and may be changed to dollars and cents by putting the point (.) *two* places from the *right,* and prefixing the sign ($).

	(16.)	(17.)	(18.)	(19.)
Multiply	$43.72	$136.04	87 cents.	$2.06
By	8	7	9	6
Product	$349.76	$952.28	$7.83	$12.36

20. At 6 cents a pound, what cost 675 pounds of rice?

21. At $4.37 a yard, what is the cost of 7 yards of cloth?

22. At $124.50 an acre, what will 5 acres of land cost?

23. What is the cost of 8 building lots, at $2015 each?

ORAL EXERCISES.

106. 1. 9 times $12 are $108. Which number is the Multiplicand? The Multiplier? The Product?

2. If 6 men can build a wall in 7 days, in how many days can 1 man build it?

ANALYSIS.—It will take 1 man 6 times as many days as it will 6 men, to build the wall; and 6 times 7 days are 42 days. Hence it will take 1 man 42 days.

3. If 7 men can do a piece of work in 10 days, how many days will it take 1 man to do the same work?

4. How many horses will consume as many bushels of oats in 1 day as 7 horses will consume in 5 days?

5. If 3 barrels of flour last 9 persons 4 months, how long will the same quantity of flour last 1 person?

6. If a man earn $18 a week, and spend $9 for board and other expenses, how much can he save in 8 weeks?

7. If Henry earn $5 a week, and James $4, how much will both earn in 7 weeks?

8. What is the difference in the cost of 6 yards of ribbon at 9 cents a yard, and 6 yards at 11 cents a yard?

9. What will be the cost of 6 cows at $26 each?

ANALYSIS.—Six cows will cost 6 times $26. 6 times 6 units are 36 units, or 3 tens and 6 units, and 6 times 2 tens are 12 tens, which plus 3 tens and 6 units, are 15 tens and 6 units, or 156. Hence 6 cows will cost $156.

10. What cost 7 pounds of figs, at 23 cents a pound?

11. What cost 8 pounds of coffee, at 42 cents a pound?

12. At $36 a ton, what will 6 tons of guano cost?

13. At $18 a barrel, what will 9 barrels of pork cost?

14. At $5 a barrel, what are 33 barrels of apples worth?

15. At $7 a week, what is the cost of 21 weeks board?

16. What cost 20 pounds of beef, at 12 cents a pound?

17. Two men start from the same place, and travel in opposite directions, one at the rate of 6 miles an hour, the other, 8 miles an hour. How far apart will they be at the end of 6 hours? 8 hours? 9 hours?

18. A woman sold a grocer 5 pounds of butter at 30 cents a pound, and received in payment 12 pounds of sugar at 9 cents a pound. How much was still due?

Find the *second member* of the following equations:

19. $20+12-\overline{3\times6}=?$
20. $16-7+(4\times0)=?$
21. $(7\times12)-(6\times11)=?$
22. $60-(0\times12)+15=?$
23. $20\times3+(40-\overline{7\times5})=?$
24. $(3\times0)+(4\times7)=?$
25. $(55-7)\div\overline{20-8}=?$
26. $(7+5)+(6+4)=?$
27. $14\times0+\overline{45-15}=?$
28. $100-\overline{12\times7}+\overline{20-4}=?$

WRITTEN EXERCISES.

107. When the multiplier consists of two or more orders of units.

1. Multiply 678 by 46.

OPERATION.

Multiplicand	678	
Multiplier	46	
1st Partial Prod.	4068	$= 678\times6$
2d Partial Prod.	2712	$= 678\times40$
Entire Prod.	31188	$= 678\times46$

ANALYSIS.—Write the numbers as before.

Since 46 is composed of 6 units and 4 tens, 46 times any number is equal to 6 times the number, *plus* 4 tens, or 40 times the number.

6 times 678 is 4068, the first *partial* product. 4 tens times 8 units are 32 *tens*, or 3 hundreds and 2 tens. Write the 2 tens in the tens' place, in the second *partial* product, and reserve the 3 hundreds to add to the product of hundreds.

4 tens times 7 tens are 28 *hundreds*, plus 3 hundreds reserved, are 31 hundreds, or 3 thousands and 1 hundred. Write the 1 hundred in the hundreds' place in the second *partial* product, and reserve the 3 thousands to add to the product of thousands.

4 tens times 6 hundreds are 24 *thousands*, plus 3 thousands reserved, are 27 thousands, or 2 tens of thousands and 7 thousands, which write in the second *partial* product. The sum of the *partial* products is the *entire product* 31188.

In multiplying, as in adding, the pupil should be required to perform the operations mentally, without moving the lips, and then to write the results.

In like manner, multiply

2. 473 by 27.
3. 738 by 35.
4. 609 by 56.
5. $36.45 by 34 ; by 47.
6. $70.65 by 55 ; by 64.
7. $29.07 by 76 ; by 82.

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

When the multiplier consists of one figure.

II. *Begin at the right and multiply the units of each order of the multiplicand by the multiplier. Write in the product the units of each result, and reserve the tens to add to the next result.*

When the multiplier consists of more than one figure.

III. *Multiply the multiplicand by the units of each order of the multiplier successively, beginning at the right, and write the right-hand figure of each partial product under the order of the multiplier used.*

The sum of the partial products is the required product.

PROOF.—*Review the work carefully, or multiply the multiplier by the multiplicand ; if the results are the same, the work is probably correct.*

When there are ciphers in the multiplier, multiply by the significant figures only, since the product of any number by 0 is 0.

8. Multiply 6432 by 75 ; by 67 ; by 136.
9. Multiply 23072 by 128 ; by 243 ; by 307.
10. Multiply $420.06 by 204 ; by 666 ; by 408.

What is the value

11. Of 67 hogsheads of sugar, at $37.75 a hogshead ?
12. Of 2347 acres of land, at $136 an acre ?
13. Of 64 horses, at $219.75 each ?

14. What will be the cost of building a line of telegraph 274 miles long, at $967 a mile?

15. If 1049 pounds of seed cotton be raised from an acre of land, how many pounds will 386 acres produce?

16. If a cotton mill manufactures 628 yards of cloth in a day, how many yards can it make in 297 days?

What is the product

17. Of 2572 bushels by 94?
18. Of $403.06 by 127?
19. Of 86072 pounds by 208?
20. Of 316 times $487.46?
21. Of 507 times 30975 days?
22. Of 325 times 6408 cents?
23. Of 370607 by 4071?
24. Of 600326 by 2645?
25. Of 730096 by 5006?
26. Of 2407068 by 3406
27. Of 408091 by 2407?
28. Of 73069 by 46035?

29. Find the cost of 386 railway coaches, at $7034.75 each.

30. What cost 802 tubs of butter, at $27.08 each?

31. 236 × 63 × 28 = ?
32. 439 × 0 × 142 = ?
33. 1927 × 613 × 802 = ?
34. 4605 × 2034 × 576 = ?

35. How many yards of shirting in 49 bales, each bale containing 26 pieces, and each piece 57 yards?

36. What is the cost of 128 barrels of beef, each containing 216 pounds, worth 13 cents a pound?

37. Three schooners, each, ship 239 cords of wood, and a fourth ships 248 cords. What is the value of the whole at $4.25 a cord?

38. If it require 108 tons of iron rail for 1 mile of track, how many tons will be required for 476 miles, and what will be its value at $145 a ton?

39. A crop of cotton was put up in 472 bales, the average weight of which was 588 pounds. What was the weight of the whole crop, and its value at 18 cents a pound?

108. To multiply by the factors of a number.

The ***Factors*** of a number are the numbers which multiplied together will produce it. Thus, 6 and 7 are factors of 42; 2, 4, and 5 are factors of 40.

The pupil should be carefully taught to distinguish between *factors* and *parts* of a number. The *factors* are *multiplied*, but the *parts* are *added*, to produce a number.

Thus, 2 and 9, 3 and 6, 2, 3, and 3, are factors of 18; but the *parts* of 18 are 9 and 9, 10 and 8, 6 and 12, 7 and 11, etc.

109. PRINCIPLE.—*The product of any number of factors will be the same in whatever order they are multiplied.*

1. Multiply 468 by 36.

OPERATION.

$$36 = 6 \times 6, \text{ or } 9 \times 4, \text{ or } 12 \times 3.$$

$$\begin{array}{rcrcrcr}
468 & \qquad & 468 & \qquad & 468 & \qquad & 468 \\
\underline{36} & & \underline{6} & & \underline{9} & & \underline{12} \\
2808 & & 2808 & & 4212 & & 5616 \\
\underline{1404} & & \underline{6} & & \underline{4} & & \underline{3} \\
16848 & & 16848 & & 16848 & & 16848
\end{array}$$

It will be observed that the multiplicand, multiplied by the given multiplier, or by any *set of factors* into which it can be separated, produces the same result.

In like manner, multiply

2. $73.04 by $48 = 8 \times 6$.
3. 50076 by $72 = 6 \times 4 \times 3$.
4. 46502 by $84 = 7 \times 4 \times 3$.
5. $206.14 by $96 = 4 \times 4 \times 6$.
6. $780.91 by 108.
7. 140086 by 120.
8. 380509 by 144.
9. $457.52 by 240.

RULE.—I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of the factors, the resulting product by another factor, and so continue until all the factors have been used.*

The last product will be the product required.

10. What will 56 acres of land cost, at $164.50 an acre?

11. At 28 cents a pound, what will be the cost of 24 sacks of coffee, each containing 64 pounds?

12. What is the value of 107 pieces of cloth, each piece containing 42 yards, at $4.28 a yard?

110. When either the multiplicand or multiplier, or both, have ciphers on the right.

1. Multiply 286 by 100.

OPERATION.

$$\begin{array}{r} 286 \\ 100 \\ \hline 28600 \end{array}$$

ANALYSIS.—Since removing a figure *one place* to the *left*, increases its value *ten times* (**44**), annexing a cipher to a number multiplies it by 10; annexing *two* ciphers multiplies it by 100, etc. Hence 286 × 100 = 28600, the product required.

2. Multiply 3240 by 600.

OPERATION.

$$\begin{array}{r} 3240 \\ 600 \\ \hline 1944000 \end{array}$$

ANALYSIS.—3240 = 324 × 10, and 600 = 6 × 100. First multiply together the two factors 324 and 6, and then multiply their product 1944, by 10 × 100, or by 1000, by annexing *three* ciphers, which gives 1944000, the required product.

What is the product

3. Of 372 by 10? By 100? By 1000? By 10000?

4. Of 860 by 50? By 400? By 1500? By 3000?

RULE.—*To the product of the significant figures, annex as many ciphers as there are ciphers on the right of either or of both of the factors.*

What is the product

5. Of $4.72 by 100?

6. Of $30.40 by 60?

7. Of $1200 by 700?

Find

8. 120 times 5000.

9. 600 times 21000.

10. 1000 times 104000.

11. $42030090 \times 3020 = ?$

12. $7000600 \times 50040 = ?$

13. There are 640 acres in 1 square mile. How many acres in 150 square miles? In 200? In 420?

14. The salary of the president is $50000 a year. How much does he receive in 8 years?

REVIEW.

ORAL EXAMPLES.

111. 1. The sum of $8+12+16$ equals the product of $9 \times$ what number?

2. The sum of $40-14$ and $12+4$ equals $7 \times$ what number?

3. The difference between $35+15$ and $24-10$ is equal to the product of what two factors? Three factors?

4. The product of what two factors is equal to the sum of 9, 20, and 11?

5. The product of 8 times 9 is equal to 6 times what number?

6. The sum of 25, 13, 8, and 10 is equal to the product of what three factors?

7. What is the sum of 3 times $\overline{3 \times 4}$, and 5 times $\overline{4 \times 3}$? What is the difference?

8. What is the product of $15+\overline{24-14}$ by $\overline{16-12}$?

9. Which is greater, $9 \times \overline{13-6}$, or 12 times $8-20$?

10. How much less is $60-(5 \times 8)$ than $16+14-10$?

11. Charles is twice as old as George, and George is 12 years old. What is the sum of their ages?

12. What is the cost of 4 brooms at 30 cents each, and 6 pounds of sugar at 11 cents a pound?

13. Mary had 18 cents, and Belle had 3 times as many less 9 cents. How many had both?

14. A young man earned $9 a week, and spent $5 a week for board. How much did he save in 12 weeks?

15. A woman sold a grocer 4 dozen of eggs at 24 cents a dozen, and received in payment half a pound of tea worth 50 cents, and 2 pounds of sugar at 11 cents a pound. How much was still due her?

16. A boy bought a book for 36 cents, a slate for 20 cents, and a pencil for 4 cents. How much change should he receive for a 1 dollar bill?

17. A lady bought 9 yards of silk at $3 a yard, 3 pairs of kid gloves at $2 a pair, 4 pairs of hose at half a dollar a pair. She gave in payment 4 ten dollar bills. How much change should she receive?

Find the *required term* in the following equations:

18. $\overline{19-7}+\overline{28-11}=?$
19. $8\times 9-16=7\times ?$
20. $21+\overline{6\times 7}=40+?$
21. $(10\times 12)-(9\times 11)=?$
22. $75-(5\times 12)=35-?$
23. $(44+19)-(50-23)=?$
24. $\overline{7\times 12}-\overline{25\times 0}=?$
25. $3\times 0\times 5+16=2\times ?$
26. $\overline{28+12}-\overline{6\times ?}=16$
27. $9\times 12+10=120-?$
28. $42-20+14=?\times 9$
29. $8+55-(?\times 8)=7$

112. By a little practice numbers containing *three* or *four* figures may be multiplied *mentally*, by first multiplying the highest order of units, and adding the product of each lower order as found.

1. Multiply 324 by 2.

Operation.—2 times 3 hundreds are 600; 2 times 2 tens are 4 tens, or 40, and 600 + 40 are 640; 2 times 4 are 8, and 640 + 8 are 648.

Omitting all but results, the required product will be easily and promptly obtained by a strictly mental process. Thus, 600, 640, 648.

In like manner, find the product of

2. 3 times 230.
3. 3 times 342.
4. 4 times 150.
5. 4 times 425.
6. 6 times 241.
7. 5 times 615.
8. 234 × 2.
9. 501 × 3.
10. 255 × 4.

WRITTEN EXAMPLES.

113. 1. If I receive $1500 salary, and pay $370 for board, $281.50 for clothing, $112.75 for books, and $196.65 for other expenses annually, what can I save in 3 years?

2. A merchant bought 7 hogsheads of sugar at $46.45 a hogshead, and sold it for $53.62 a hogshead. How much did he gain?

3. Paid $2709 for 388 barrels of flour, and sold the same at $9.12 a barrel. How much was the gain?

4. If a man have an income of $5670 a year and his daily expenses average $7.25, how much can he save in a year of 365 days?

5. What sum must be added to 272 × 400, to make the amount 126720?

6. What is the difference between 40706 — (308 × 56), and (97 × 340) — 12400?

7. Multiply $98+\overline{6\times(37+50)}$ by $(\overline{64-50}\times 5)-10$.

8. Multiply $675-(77+56)$ by $(3\times 155)-(214-28)$.

9. A man owing $15760, gave in payment 5 lots of land, worth $730 each, 5 horses, valued at $236.50 each, an interest he had in a coal mine worth $2000, and $1728.75 in money. How much remained unpaid?

10. A farm-house is worth $3246, the farm is worth 3 times as much plus $1200, and the stock is worth twice as much as the house, less $1875. What is the value of the whole, and of the farm and stock?

11. What is the difference in the cost of 48 horses at $184.50 each, and 130 sheep at $4.80 a head?

12. Bought 150 barrels of flour for $1150, and finding 25 barrels of it worthless, sold the remainder at $9 a barrel. Did I gain or lose, and how much?

Complete the following equations:

13. $(142 + 405) \times (1000 - 850) - 5000 = ?$

14. $(97 \times 1000) - (75 \times \overline{500 - 420}) + 1500 = ?$

15. $\$73.46 - (\$.94 + \$3.02) + (\$47 \times 35) = ?$

16. $(\$246.08 \times 104) + (\$2000 - \$240.50) \times 10 = ?$

114. SYNOPSIS FOR REVIEW.

MULTIPLICATION.

- 1. DEFINITIONS.
    - 1. Multiplication. 2. Multiplicand. 3. Multiplier. 4. Product. 5. Sign of Multiplication.
- 2. PRINCIPLES, 1 and 2.
- 3. RULE; I, II, III.
- 4. PROOF.
- 5. WHEN EITHER FACTOR CONTAINS CENTS.
- 6. BY FACTORS.
    - 1. Definition of factors.
    - 2. Principle.
    - 3. Rule.
- 7. MULTIPLIER AND MULTIPLICAND.
    - 1. When one or both have ciphers on the right.
    - 2. Rule.

DIVISION

ORAL EXERCISES.

115. 1. How many 4's are 12? Are 16? Are 24?

2. How many lots, of 5 acres each, in 20 acres?

3. How many 5's in 15? In 30? In 35? In 50?

4. How many barrels, each holding 3 bushels, will be required for 18 bushels of apples? 21 bushels?

5. How many times can 6 yards of cloth be taken from a piece containing 30 yards.

6. How many times can 6 cents be taken from 23 cents, so as to have 5 cents remaining?

7. Distribute $28 equally among 7 men. How many dollars will each receive?

Do you find how many times 7 men are contained in $28, or do you find *one* of 7 equal parts of $28?

8. How do you find one of 8 equal parts of a number? Of 9 equal parts? Of 6 equal parts?

9. What is one of 4 equal parts of 40? Of 36? Of 48?

10. What is one of 6 equal parts of 30? Of 42? Of 48?

11. What is one of 7 equal parts of 56 pounds?

12. How many times 8 cents are 48 cents? Is the result a concrete or an abstract number?

13. What is one of 8 equal parts of 48 cents? Is the result a concrete or an abstract number?

DEFINITIONS.

116. ***Division*** is the process of finding *how many times* one number is contained in another of the same kind, or of finding *one* of the *equal parts* of a number.

117. The ***Dividend*** is the number to be divided.

118. The ***Divisor*** is the number by which to divide.

119. The ***Quotient*** is the result of the division, and shows *how many times* the dividend contains the divisor.

The division is said to be *exact* when there is no remainder.

The part of the dividend remaining when the division is not *exact* is called the *Remainder*, and must always be *less* than the divisor.

120. The ***Sign of Division*** is ÷. It is read *divided by.*

It shows that the number before which it is placed is to be divided by the one after it; thus, 54 ÷ 9 is read 54 *divided by* 9.

121. Division is also indicated by placing the dividend above the divisor with a line between them; thus, $\frac{72}{8}$ is read 72 *divided by* 8.

122. PRINCIPLES.—In finding *how many times* one number is contained in another:

1. *The divisor and dividend are like numbers, and the quotient an abstract number.*

In finding one of the *equal parts* of a number:

2. *The dividend and quotient are like numbers, and the divisor an abstract number.*

3. *The dividend is equal to the product of the divisor by the quotient, plus the remainder.*

ORAL EXERCISES.

123. $36 \div 9 = ?$ $63 \div 9 = ?$ $64 \div 8 = ?$ $84 \div 7 = ?$
$42 \div 7 = ?$ $56 \div 8 = ?$ $66 \div 6 = ?$ $72 \div 12 = ?$
$40 \div 5 = ?$ $45 \div 5 = ?$ $72 \div 9 = ?$ $96 \div 8 = ?$

$\frac{63}{7} = ?$ $\frac{96}{8} = ?$ $\frac{77}{7} = ?$ $\frac{120}{10} = ?$
$\frac{72}{9} = ?$ $\frac{70}{7} = ?$ $\frac{96}{12} = ?$ $\frac{108}{12} = ?$
$\frac{48}{12} = ?$ $\frac{84}{12} = ?$ $\frac{72}{6} = ?$ $\frac{132}{11} = ?$

1. Divide by 2, from 2 in 2 to 2 in 24.

OPERATION.—2 in 2, once; 2 in 4, twice; 2 in 6, 3 times; 2 in 8, 4 times; 2 in 10, 5 times, and so on to 2 in 24, 12 times.

In the same manner, divide

2. By 3, from 3 in 3, to 3 in 36.
3. By 4, from 4 in 4, to 4 in 48.
4. By 5, from 5 in 5, to 5 in 60.
5. By 6, from 6 in 6, to 6 in 72.
6. By 7, from 7 in 7, to 7 in 84.
7. By 8, from 8 in 8, to 8 in 96.
8. By 9, from 9 in 9, to 9 in 108.
9. By 10, from 10 in 10, to 10 in 120.

The pupil may *reverse* the above; thus, 2 in 24, 12 times; 2 in 22, 11 times; 2 in 20, 10 times, and so on.

Also *combine* the two; thus, 3 in 3, once; 3 in 6, twice, 2 in 6, 3 times; 3 in 12, 4 times, 4 in 12, 3 times; and so on to 3 in 36, 12 times, 12 in 36, 3 times

124. Division may also be regarded as a short method of performing several *subtractions* of a number.

Thus, $24 - 6 = 18$; $18 - 6 = 12$; $12 - 6 = 6$; $6 - 6 = 0$. We have performed *four* subtractions of 6, hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

125. Since one number is contained in another as many times as it is a *factor* of the other, division may be regarded as the *reverse* of multiplication.

In Multiplication, *both factors* are given to find the *product;* in Division, *one factor* and the *product* (answering to the *dividend*) are given to find the *other factor,* which answers to the *quotient.*

Thus, $6 \times 4 = 24$, 6 being a factor of 24, *four* times; hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

126. The Object of Division is *twofold.*

First. To find *how many times* one number is contained in another of the *same kind.*

Ex. At 5 cents each, how many pencils can be bought for 20 cents.

Since 5 cents taken 4 times equals 20 cents ($5 \times 4 = 20$), it follows that 5 cents is contained in 20 cents 4 times.

ANALYSIS.—As many pencils can be bought for 20 cents, as 5 cents are contained times in 20 cents, which are 4 times. Hence, etc.

127. Second. To *separate* a given number into as many *equal parts* as there are units in another.

Ex. If 4 pencils cost 20 cents, what is the cost of 1 pencil?

Since 5 cents taken 4 times equals 20 cents, it follows that 5 cents is one of the four *equal parts* of 20 cents ($5 + 5 + 5 + 5 = 20$), and we say *one-fourth* of 20 cents is 5 cents.

ANALYSIS.—Since 4 pencils cost 20 cents, 1 pencil costs *one-fourth* of 20 cents, which are 5 cents. Hence 1 pencil costs 5 cents.

128. The *equal parts* into which a unit or whole thing is divided are sometimes called *fractions.*

129. The *names* of these equal parts of a unit vary according to the *number* of these parts; thus, *one-half* is one of *two* equal parts, *one-third* is one of *three* equal parts into which the whole thing or number is divided.

So in like manner we have *fourths, fifths, sixths, sevenths, eighths, tenths, twelfths, twentieths,* etc.

130. These parts are expressed by writing the number denoting the *name* of the parts below a short horizontal line as a *divisor,* and the *number* of parts taken or used, above the line as a *dividend.*

Thus, $\frac{1}{2}$, signifies 1 *divided by* 2, and is read, *one-half.*

$\frac{2}{3}$, signifies 2 *divided by* 3, and is read *two-thirds.*

$\frac{7}{12}$, signifies 7 *divided by* 12, and is read, *seven-twelfths,* etc.

ORAL EXERCISES.

131. 1. If a number is separated into *two* equal parts, what is each part called?

Ans. ***One-half*** of the number, written $\frac{1}{2}$.

2. If $18 are equally divided between two poor families, how much does each receive? What *part* of the whole?

3. What is *one-half* of 12? Of 16? Of 20? Of 24?

4. If a number is separated into *three* equal parts, what is each part called? ***One-third*** of the number, $\frac{1}{3}$.

5. If 15 peaches are equally distributed among 3 boys, what *part* of the whole will each receive?

6. What is *one-third* of $15? Of 21 days? Of 30 rods?

7. Divide an acre of land into *four* equal parts. What is one of the parts called? ***One-fourth*** of an acre, $\frac{1}{4}$.

8. What are 2 of the parts called? ***Two-fourths,*** $\frac{2}{4}$. Three of the parts? ***Three-fourths,*** $\frac{3}{4}$.

9. If 48 marbles are given to 4 boys, to each an equal number, what *part* of the whole does 1 boy receive? Two boys? Three boys? How many marbles?

10. What is *one-fourth* of $24? Of 48 miles?

11. If a number is divided into *five* equal parts, what is each part called? ***One-fifth*** of the number, $\frac{1}{5}$. Two parts? ***Two-fifths***, $\frac{2}{5}$.

12. If $20 are paid for 5 barrels of apples, what *part* of $20 is paid for 1 barrel? For 2 barrels? For 3 barrels?

13. What is $\frac{1}{5}$ of 30? Of $40? Of 45 rods?

14. If a number is divided into *six* equal parts, what is each part called? ***One-sixth*** of the number, $\frac{1}{6}$.

15. If into *seven* equal parts? ***One-seventh***, $\frac{1}{7}$.

16. If into *eight* equal parts? ***One-eighth***, $\frac{1}{8}$.

17. If into *nine* equal parts? ***One-ninth***, $\frac{1}{9}$.

18. If into *ten* equal parts? ***One-tenth***, $\frac{1}{10}$.

19. If into *twelve* equal parts? ***One-twelfth***, $\frac{1}{12}$.

20. Find one-half of 2, one-half of 4, one-half of 6, one-half of 8, and so on to one-half of 20.

21. Find one-third of 3, one-third of 6, one-third of 9, one-third of 12, and so on to one-third of 30.

22. Find $\frac{1}{4}$ of 4, $\frac{1}{4}$ of 8, $\frac{1}{4}$ of 12, $\frac{1}{4}$ of 16, to $\frac{1}{4}$ of 40.

23. Find $\frac{1}{5}$ of 5, $\frac{1}{5}$ of 10, $\frac{1}{5}$ of 15, $\frac{1}{5}$ of 20, to $\frac{1}{5}$ of 50.

24. Find $\frac{1}{6}$ of 6, $\frac{1}{6}$ of 12, $\frac{1}{6}$ of 18, $\frac{1}{6}$ of 24, to $\frac{1}{6}$ of 60.

25. Find $\frac{1}{7}$ of 7, $\frac{1}{7}$ of 14, $\frac{1}{7}$ of 21, $\frac{1}{7}$ of 28, to $\frac{1}{7}$ of 70.

26. Find $\frac{1}{8}$ of 8, $\frac{1}{8}$ of 16, $\frac{1}{8}$ of 24, $\frac{1}{8}$ of 32, to $\frac{1}{8}$ of 80.

27. Find $\frac{1}{9}$ of 9, $\frac{1}{9}$ of 18, $\frac{1}{9}$ of 27, $\frac{1}{9}$ of 36, to $\frac{1}{9}$ of 90.

28. Find $\frac{1}{10}$ of 10, $\frac{1}{10}$ of 20, $\frac{1}{10}$ of 30, to $\frac{1}{10}$ of 100.

29. How do you find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of any number?

30. How many yards of cloth, at $4 a yard, can be bought for $36?

ANALYSIS.—As many yards as $4 are contained times in $36, which are 9 times. Hence 9 yards can be bought for $36.

31. At $6 a ton, how many tons of coal can be bought for $24? For $30? For $54? For $72?

32. If 7 cords of wood cost $42, what does 1 cord cost?

Analysis.—Since 7 cords of wood cost $42, 1 cord costs 1 *seventh* of $42, or $6. Hence 1 cord costs $6.

33. A man sold 8 bushels of cranberries for $32. How much did he receive a bushel for them?

34. A farmer gathered 108 bushels of apples from 9 trees. What was the average number of bushels to each tree?

35. A merchant paid $96 for 8 pieces of dress goods. What was the cost of each piece?

36. If a farm of 120 acres is divided into 12 equal lots, how many acres does each lot contain?

WRITTEN EXERCISES.

132. When the divisor consists of but one order of units.

1. Divide 875 by 7.

OPERATION.

```
Divisor. Dividend. Quotient.
     7)875(125
       7
       ---
       17
       14
       ---
        35
        35
        --
```

Analysis.—Write the divisor at the left of the dividend with a line between them.

7 is contained in 8 hundreds, 1 hundred times, with a remainder. Write the 1 hundred at the right of the dividend, for the first figure of the quotient. Multiply the divisor 7 by the 1 hundred of the quotient, and write the product, 7 hundreds, under the hundreds of the dividend. Subtract, and to the remainder 1 hundred, annex the 7 tens of the dividend, making 17 tens.

7 is contained in 17 tens, 2 tens times, with a remainder. Write the 2 tens in the quotient. Multiply the divisor 7 by the 2 tens, and subtract the product from the partial dividend, 17 tens. To the remainder 3 tens, annex the 5 units of the dividend, making 35 units.

7 is contained in 35 units, 5 times, which write in the quotient. Multiplying and subtracting as before, nothing remains. Hence, etc.

The solution of the preceding example may be abbreviated by what is termed *Short Division,* as follows:

OPERATION.
7)875
Quotient 125

ANALYSIS.—7 is contained in 8, once, and 1 remainder. 1 prefixed to 7 makes 17. 7 is contained in 17, 2 times and 3 remainder. 3 prefixed to 5 makes 35, and 7 is contained in 35, 5 times. Hence the quotient is 125.

133. In ***Short Division*** only the quotient is written, the operations being performed *mentally.* It is generally used when the divisor does not exceed 12.

In like manner, divide and analyze the following:

(2.)	(3.)	(4.)	(5.)
6)7944	7)9464	8)8928	5)6895

6. Divide 92352 by 8; by 6; by 4.
7. Divide 83762 by 7; 79880 by 6; 3263 by 8.

OPERATION.
8)3263
Quotient 407$\frac{7}{8}$

ANALYSIS.—Since 8 is not contained in 3 thousands, unite the 3 thousands and 2 hundreds, making 32 hundreds. 8 is contained in 32 hundreds, 4 hundreds times, which write in the hundreds' place in the quotient. Next, 8 is not contained in 6 tens, so write a cipher in tens' place in the quotient, and unite the 6 tens and 3 units. 8 is contained in 63 units 7 times and 7 units remainder, which write over the divisor and add as a part of the quotient. Hence the quotient is 407$\frac{7}{8}$.

PROOF.—Multiply the quotient 407 by the divisor 8, and the product is 3256; 3256 plus the remainder 7, equals the dividend 3263. (PRIN. 3.)

8. Divide 8135464 by 6; by 8; by 7; by 5; by 9.
9. Divide $48.56 by 8 cents.

OPERATION.
$.08)$48.56
607 times.

Eight cents may be written $.08 (**73**). When the divisor and dividend are like numbers, the quotient is an abstract number (PRIN. 1). Hence 8 cents are contained in $48.56, 607 *times.*

10. Divide $48.56 by 8.

OPERATION.
8)$48.56
$6.07

When the divisor is an abstract number, the dividend and quotient are like numbers (PRIN. 2). Hence 1 *eighth* of $48.56 is $6.07.

Solve and prove,

(11.)	(12.)	(13.)	(14.)
9)$217.62	7)$6.44	$7)$644	$.07)$6.44
$24.18	$.92	92 times.	92 times.

How many times

15. Are $8 contained in $15096? In $58424? In $23064?

16. Is 7 contained in 330457? In 19278? In 918271?

17. Is 9 contained in 436281? In 605675? In 1039126?

Find

18. 1 *fifth* of $863.25.

19. 1 *sixth* of 34807 tons.

20. 1 *eighth* of 20673 days.

21. 1 *ninth* of $7384.50.

What is

22. $\frac{1}{7}$ of 500322 miles?

23. $\frac{1}{4}$ of 32876 men?

24. $\frac{1}{3}$ of 60349 acres?

25. $\frac{1}{12}$ of 760344 rods?

26. How many barrels of flour at $8 a barrel, can be bought for $12736? For $7068?

27. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

28. If 9 acres of land cost $976.50, what is the cost of 1 acre?

29. How many oranges can be bought for $3.72, at 4 cents a piece?

30. At 8 cents a yard, how many yards of ribbon can be bought for $7.28?

31. Paid $1792 for 7 horses. What did each cost?

ORAL EXERCISES.

134. 1. The quotient of two numbers is 15, and the divisor is 8. What is the dividend?

2. The dividend is 96, and the quotient is 6. What is the divisor?

3. The quotient is 12, the remainder is 9, and the divisor is 11. What is the dividend?

4. If 12 yards of cloth cost $35, for how much a yard must it be sold to gain $13?

5. A man received $50 for 5 barrels of pears, and paid all but $14 for 4 chairs. What did each chair cost?

6. If 4 weeks' board cost $28, what will 9 weeks' board cost?

ANALYSIS.—One week's board will cost 1 *fourth* of $28, or $7; and 9 weeks' board will cost 9 times $7, or $63. Hence, etc.

7. If 8 yards of silk cost $32, what will 12 yards cost?

8. What will 15 sheep cost, if 5 sheep cost $35?

9. How many cords of wood at $4 dollars a cord, will pay for 6 barrels of flour at $8 a barrel?

ANALYSIS.—Six barrels of flour will cost 6 times $8, or $48; and $4, the price of 1 cord of wood, are contained in $48, 12 times. Hence, etc.

10. How many days' labor at $4 a day will pay for 3 tons of coal at $6 a ton, and 2 tons of hay at $15 a ton?

11. How many pounds of meat at 12 cents a pound, will cost as much as 9 pounds of cheese at 8 cents a pound?

Complete the following equations:

12. $(8 \times 0) + (6 \times 4) \div 8 = ?$
13. $\overline{10 \times 12} - \overline{0 \times 6} \div 6 = ?$
14. $(9 \times 11) - \overline{54 \div 6} + 20 = ?$
15. $\overline{63 \div 7} \times 0 + 12 = ?$
16. $(108 \div 12) \times 11 - 25 = ?$
17. $\overline{90 - 18} \div (44 - \overline{7 \times 6}) = ?$

WRITTEN EXERCISES.

135. When the divisor consists of more than one order of units.

1. Divide 5437 by 26.

OPERATION.

Divisor. Dividend. Quotient.

$$\begin{array}{l}
26)5437(209\tfrac{3}{26} \\
\underline{52} \\
237 \\
\underline{234} \\
3 \text{ Remainder.}
\end{array}$$

ANALYSIS.—26 is contained in 54 hundreds, 2 hundred times, with a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 26 by this quotient figure, and subtract the product, 52 hundreds, from 54 hundreds, the *first* partial dividend, and there remains 2 hundreds. To this annex the 3 tens of the dividend, making 23 tens for the *second* partial dividend.

26 is not contained in 23, so write a cipher in the quotient and bring down the 7 units of the dividend, making 237 units for the *third* partial dividend.

26 is contained in 237 units 9 times, with a remainder. Write the 9 units in the quotient, and multiplying and subtracting as before, there remain 3 units, which write over the divisor, and annex as a part of the quotient. Hence the quotient is $209\frac{3}{26}$.

136. ***Long Division*** is the process of dividing in which the steps of the solution *are all written.*

2. Find how many times 204 is contained in 1041835.

OPERATION.

Divisor. Dividend. Quotient.

$$\begin{array}{l}
204)1041835(5107\tfrac{7}{204} \\
\underline{1020} \\
218 \\
\underline{204} \\
1435 \\
\underline{1428} \\
7 \text{ Remainder.}
\end{array}$$

PROOF.

$$\begin{array}{rl}
5107 & \text{Quotient.} \\
\underline{204} & \text{Divisor.} \\
20428 & \\
\underline{10214} & \\
1041828 & \\
\underline{7} & \text{Remainder.} \\
1041835 & \text{Dividend.}
\end{array}$$

3. Divide 32762 by 14; by 16; by 23; by 28.

4. Divide 130426 by 58; by 63; by 81; by 74.

RULE.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the least number of the left hand orders of the dividend that will contain it, and write the result for the first figure of the quotient.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the figure of the next lower order of the dividend for a new partial dividend, and divide as before.*

IV. *Proceed in the same manner until all the orders of the dividend have been used.*

V. *If any partial dividend does not contain the divisor, write a cipher in the quotient, and annex the next order of the dividend, and proceed as before.*

VI. *If there is at last a remainder, write it, with the divisor underneath, as a part of the quotient.*

PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will be equal to the dividend.*

1. If the product of the divisor and quotient be *greater* than the *partial* dividend, the quotient is *too large*, and must be *diminished.*
2. If any remainder is *equal to* or *greater* than the divisor, the quotient is *too small* and must be *increased.*

137. When the divisor and dividend are both concrete numbers, they must be of the *same name*. Hence, if one be *dollars*, and the other *cents* or dollars and cents, before dividing, change so that both may be *cents*.

138. Since 100 cents make 1 dollar, there are 100 times as many cents as dollars. Hence,

To change a number representing dollars to a number representing cents, annex two ciphers (110), omit the sign ($) and write the word *cents* after it.

To change dollars and cents to the same form, omit the sign ($) and the point (.) and write the word *cents* at the right.

5. Divide $46.92 by 23.

OPERATION.

23)$46.92($2.04
46
92
92

6. Divide $46.92 by 23 cents.

OPERATION.

23)4692(204 times.
46
92
92

7. Divide $46.92 by $23.

OPERATION.

2300(4692(2 $\frac{92}{2300}$ times.
4600
92

8. Divide $46 by 23 cents.

OPERATION.

23)4600(200 times.
46
00

In like manner divide, and prove the following:

9. $325.72 by 34.
10. $938.07 by 63.
11. $3176.46 by 126.
12. $49.56 by 14 cents.
13. $87.36 by 21 cents.
14. $288.96 by $.43.
15. $810.98 by $.46.
16. $594 by 18 cents.
17. $1385 by $105.
18. $32.48 by $7.

How many times

19. Is 47 contained in 30176? In 27865? In 103474?

20. Is 185 contained in 200376? In 4701625?

21. The annual receipts of a company are $570685. What is the average a day, if there are 313 working days?

22. If 867 shares of railroad stock are valued at $84099, what is the value of each share?

23. A plantation of 736 acres was sold for $55936. What was the price of an acre?

24. Paid $17100 for a farm, at the rate of $36 an acre. How many acres did it contain?

25. How many horses, at $125 each, will $4735 buy, and how much money will be left?

Divide
26. 33490 by 85.
27. 740070 by 135.
28. 1554768 by 216.
29. 5497800 by 175.
30. 3931476 by 556.
31. 5120401 by 587.
32. 1018090 by 1669.
33. 73484248 by 2624.

Divide
34. 863256 by 736.
35. 1646301 by 381.
36. 5226412 by 2567.
37. 11214887 by 3076.
38. 75862500 by 10115.
39. 313194105 by 7153.
40. 1246038849 by 269181.
41. 2331883954 by 6739549

139. To divide by the factors of a number.

1. Divide 644 by 28, using the factors.

OPERATION.

$$\begin{array}{r} 4\underline{)\,644} \\ 7\underline{)\,161} \\ 23 \end{array}$$

ANALYSIS.—Since 28 is equal to 4 times 7, either divide by 28, or by its factors 4 and 7. Now, 644÷4=161; but this quotient is 7 times too great, and must therefore be divided by 7; hence, 161÷7=23, the true quotient.

Factors.
2. Divide 6228 by 36, or by 4, and 9.
3. Divide 27360 by 96, or by 3, 4, and 8.
4. Divide 526050 by 126, or by 2, 7, and 9.
5. Divide 73416 by 168, or by 4, 6, and 7.

6. Divide 5831 by 84, using the factors, 3, 4, and 7.

OPERATION.

$3\underline{)5831}$

$4\underline{)1943}$ 2

$7\underline{)485}$ . . . $3 \times 3 =$ 9

69 . . $2 \times 4 \times 3 = \underline{24}$

True Remainder. 35

$69\frac{35}{84}$ Quotient.

ANALYSIS. — Since 84 is equal to $3 \times 4 \times 7$, divide by 84, or by its factors 3, 4, and 7.

$5831 \div 3 = 1943$, and a remainder of 2, which being a part of the dividend, is also a part of the true remainder.

$1943 \div 4 = 485$, and a remainder of 3. Since a unit of the first quotient 1943, equals 3 units of the dividend, this second remainder 3 being a part of 1943, equals 3 × 3, or 9 units of the dividend.

$485 \div 7 = 69$, and a remainder of 2. Since a unit of the second quotient 485 equals 4 units of the first quotient 1943, this third remainder 2 being a part of 485, equals 2 × 4 × 3, or 24 units of the dividend. Hence the first *partial* remainder is 2, the second is 9, the third is 24, and the *true* remainder 35 ; and the quotient $69\frac{35}{84}$.

7. Divide 139074 by 72, using its factors 3, 4, and 6.

8. Divide 7360479 by 96, using its factors 2, 6, and 8.

RULE.—I. *Separate the divisor into two or more factors.*

II. *Divide the dividend by one of these factors, and the quotient thus obtained by another factor, and so on until all the factors have been used as divisors.*

III. *If there be remainders, multiply each remainder by all the divisors preceding the one that produced it.*

IV. *Add the products and the remainder from the first division, if any, and the sum will be the true remainder.*

9. Divide 376875 by 315, or by 5, 7, and 9.

10. Divide 73522 by 135, or by 3, 5, and 9.

11. Divide 401976 by 245, or by 5, 7, and 7.

140. When the divisor has ciphers on the right.

1. Divide 4067 by 10.

OPERATION.

$$\begin{array}{l} 1|0\underline{)406}|\underline{7} \\ \quad 406\ .\ .\ 7 \text{ Rem.} \\ \quad 406\tfrac{7}{10} \text{ Quotient.} \end{array}$$

ANALYSIS.—Since removing any order of figures *one place* to the *right*, diminishes its value *ten times* (**45**), by cutting off, or taking away, the right-hand figure of a number, each of the remaining figures, being removed one place to the right, is diminished in value ten times, or *divided by* 10.

For similar reasons, cutting off *two* figures divides by 100, cutting off *three* figures, divides by 1000, and so on. The remaining figures are the quotient, and those cut off, the remainder.

Divide

2. 37684 by 100.
3. 103076 by 1000.
4. 267104 by 10000.
5. 5023001 by 100000.

6. Divide 2416700 by 6000.

OPERATION.

$$\begin{array}{l} 6|000\underline{)2416}|700 \\ \quad 402\ .\ .\ 4700 \text{ Rem.} \\ \quad 402\tfrac{4700}{6000} \text{ Quotient.} \end{array}$$

ANALYSIS.—Resolve 6000 into the factors 1000, and 6. First divide by 1000, by cutting off the three right-hand figures of the dividend. The quotient is 2416, and a remainder of 700. Next divide 2416 by 6; the quotient is 402 and a second remainder of 4 thousands, which prefixed to the *first* remainder 700 gives a *true* remainder of 4700. Hence the quotient is $402\frac{4700}{6000}$.

In like manner, divide

7. 307200 by 900.
8. 7820305 by 28000.
9. 5761321 by 2040.
10. 8073160 by 14800.

RULE.—I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

11. If it require $34400 to pay a regiment of 800 men, how much does each man receive?

12. At $3400, how many lots can be bought for $68000?

13. How many bales, each weighing 470 pounds, can be made of 39500 pounds of cotton?

GENERAL PRINCIPLES OF DIVISION.

141. The quotient depends upon the *relative* values of the dividend and divisor. Hence, any change in the value of either dividend or divisor, will produce a change in the value of the quotient. But some changes may be made upon both dividend and divisor, which will not affect their *relative* values, and consequently will not affect the quotient. To illustrate, let $54 \div 9 = 6$, be the fundamental equation, with which the following are to be compared:

1. $(54 \times 3) \div 9 = 162 \div 9 = 18$.	Multiplying the dividend by 3 multiplies the quotient by 3.
2. $54 \div (9 \div 3) = 54 \div 3 = 18$.	Dividing the divisor by 3 multiplies the quotient by 3.
3. $(54 \div 3) \div 9 = 18 \div 9 = 2$.	Dividing the dividend by 3 divides the quotient by 3.
4. $54 \div (9 \times 3) = 54 \div 27 = 2$.	Multiplying the divisor by 3 divides the quotient by 3.
5. $(54 \times 3) \div (9 \times 3) = 162 \div 27 = 6$.	Multiplying both dividend and divisor by 3 does not change the quotient.
6. $(54 \div 3) \div (9 \div 3) = 18 \div 3 = 6$.	Dividing both dividend and divisor by 3 does not change the quotient.

These six equations illustrate the following

142. GENERAL PRINCIPLES OF DIVISION.

1. *Multiplying the dividend,* or *Dividing the divisor,* } *Multiplies the quotient.*
2. *Dividing the dividend,* or *Multiplying the divisor,* } *Divides the quotient.*
3. *Multiplying or dividing both dividend and divisor by the same number,* } *Does not change the quotient.*

These *three principles* may be embraced in one

GENERAL LAW.

143. *A change in the dividend produces a LIKE change in the quotient, but a change in the divisor produces an OPPOSITE change in the quotient.*

GENERAL REVIEW.

ORAL EXAMPLES.

144. 1. The sum of three numbers is 40. One of the numbers is 12, and another is 15. What is the third ?

2. The difference of two numbers is 16, and the smaller is 12. What is the larger ?

3. The difference of two numbers is 18, and the larger is 30. What is the smaller ?

4. The product of two numbers is 132, and one of the numbers is 11. What is the other ?

5. What three numbers besides 10 will divide 120 without a remainder ?

6. The sum of two numbers is 21, and the greater 12. What is the product of the two numbers ?

7. The quotient of two numbers is 45, and the divisor 8. What is the dividend?

8. How many times can 8 bushels of grain be taken from a bin containing 52 bushels, and what will remain?

9. A news-boy sold 24 papers at 4 cents each, and thereby gained 48 cents. At what rate did he buy the papers?

10. The dividend is 240 and the quotient 12. What is the divisor?

11. The quotient is 20, the remainder 8, and the divisor 9. What is the dividend?

12. A drover bought 10 sheep at $8 a head, and sold them for $96. How much did he gain a head?

How many

13. In each of 5 equal parts of $(\overline{9 \times 12} - \overline{8 \times 6})$?

14. In each of 9 equal parts of $(56 - \overline{0 \times 7} + 16)$?

15. In each of 7 equal parts of $(72 - 40 + \overline{37 - 20})$?

16. If 5 men can build a wall in 9 days, in how many days can 3 men build it?

ANALYSIS.—It will take 1 man 5 times 9 days, or 45 days; and 3 men can build it in 1 *third* of 45 days, or 15 days. Hence, etc.

17. How long will it take 7 men to do the same work that 14 men can perform in 3 days?

18. If 9 days' work will pay for 6 tons of coal at $6 a ton, what is the price of a day's labor?

19. How much pork can be bought for 96 cents, if 9 pounds cost 72 cents?

20. If 5 men can build a wall in 8 days, how many men can build it in 4 days?

ANALYSIS.—It will require 8 times 5 men, or 40 men, to build it in 1 day, and 1 *fourth* of 40 men, or 10 men, to build it in 4 days. Hence, etc.

21. How many men will be required to do the same work in 5 days that 4 men can do in 40 days?

22. If 6 men can dig a ditch in 5 days, how many men would be required to dig it in 1 day? In 2 days? In 3 days? In 6 days? In 10 days?

23. At the rate of 24 miles in 8 hours, how many miles would a man walk in 12 hours?

24. If a woman pay 60 cents for some lemons, at the rate of 10 cents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?

25. If 5 barrels of flour are worth $60, how many cords of wood at $4 a cord will pay for 3 barrels?

26. If 12 yards of cloth cost $40, for how much must it be sold a yard to gain $20?

27. What cost 9 quarts of milk, if 4 quarts cost 24 cents?

28. How many bags will be required to hold 108 bushels of wheat, if 4 bags hold 9 bushels?

29. To 6 add 8, subtract 4, multiply by 5, add 6, divide by 8, and what is the result?

30. How much greater is 7 times 8 plus 4, than 72 divided by 9, multiplied by 7?

31. How much less is 10 times 10, diminished by 4 times 10, plus 12, than 100 divided by 10, plus 8 times 11?

Find the *required* term in the following epuations:

32. $25+9-\overline{32-4}=?$
33. $4\times12+\overline{3\times9}=?$
34. $\overline{60-12\div6}\times ?=56$
35. $\overline{72\div9}\times\overline{22-10}=?$
36. $\overline{120\div20+48}\div ?=9$
37. $\overline{96\div8}\times9=?\times12$
38. $(32+12\div11)\times ?=80$
39. $(132\div11-4)\times9=60$
40. $42+24-15=?+10$
41. $\overline{48+36}\div\overline{48-36}=16-?$
42. $120-(7\times12)\div6=?\div11$
43. $\overline{49+14}\div(28-19)=25-?$

WRITTEN EXAMPLES.

145. 1. Subtract 2520 from the sum of 3472, 450, 1254, and 56; divide the remainder by 113, and multiply the quotient by 205. What is the result?

2. How many times can 236 be subtracted from 2124?

3. How many times 236 will produce 2124?

4. The factors of a number are 36 + 114, and 5640 — 3007. What is the number?

5. The product of two numbers is 30128, and one of the numbers is 4200 ÷ 75. What is the other?

6. Divide the product of 204 and 378, by their difference.

7. What must be added to the sum of $12.36 and $7.62, to amount to $30.76?

8. What is the difference between 746 × 23 and 18975 ÷ 25?

9. A man owing a debt of $3000, paid $756.50 at one time, $1289.75 at another, and then made a third payment large enough to reduce the debt to $925.60. What was the third payment?

10. How many pounds of butter at 40 cents a pound are worth as much as 1600 bushels of oats at 75 cents a bushel?

11. If a man gain $638.75 by selling 365 barrels of flour at $9.25 a barrel, at what price did he buy it?

12. The multiplier is 36 and the product 170352; if the multiplier be 1 fourth as great, what is the product?

13. The multiplier is 204 and the multiplicand is 17605; if the multiplicand be 1 fifth as great, what is the product?

14. If a mechanic receive $1500 a year for his labor, and his expenses are $968, in what time can he save enough to buy 28 acres of land at $133 an acre?

15. The multiplier is 48, the product is 166656; if the multiplicand be 1 third as great, what is the product?

16. The divisor is 16, the quotient 12624; if the divisor be 1 fourth as great, what is the quotient?

17. The divisor is 24, and the quotient is 43950; if the divisor be increased 6 times, what will be the quotient?

18. The quotient is 91864; if the divisor be 1 ninth as great, what is the quotient?

19. A grocer bought two kinds of syrup; one for 54 cents a gallon, and the other for 62 cents. What was the *average* cost a gallon?

Operation.—(54 cents + 62 cents) ÷ 2 = 58 cents.

The *average* of *two* numbers is *one-half* their sum, the average of *three* numbers is *one-third* their sum, etc.

20. A merchant bought 3 kinds of tea, some at 60 cents, some at 78 cents, and some at 90 cents a pound. What was the average cost a pound?

21. A keeper of a toll bridge received $104 toll on Monday, $97 on Tuesday, $128 on Wednesday, and $99 on Thursday. What was the average daily receipts?

22. Sold 3 city lots for $1500, $2976, and $1895, respectively. What was the average price of each?

23. If a young man receive a salary of $25 a week, and he pays $8.75 for his board, and $4.65 for other expenses, in how many weeks can he pay a debt of $487.20?

24. A man having $4578 paid out all but $1642, in 8 weeks. What was the average amount paid out each week?

25. Bought 140 acres of land for $7560, and sold 86 acres of it at $75 an acre, and the remainder at cost. How much was gained?

26. A father gave his property to his 4 children. To the first he gave $6780, to the second $8200, to the third $1526 more than to the first, and to the fourth $1345 less than to the third. What was the value of his property?

27. The sum of two numbers is 184, and their difference is 42. What are the numbers?

ANALYSIS.—Since 184 is the sum of the numbers, if the difference 42 be subtracted from the sum 184, the remainder 142 will be *twice* the *less* number. $142 \div 2 = 71$ the less number; and $71 + 42 = 113$ the *greater* number.

Or, if the *difference* 42 be added to the *sum* 184, the amount 226, will be *twice* the *greater* number. $226 \div 2 = 113$ the greater number; and $113 - 42 = 71$ the less number.

PROOF.—$113 + 71 = 184$ the sum.

28. The sum of two numbers is 5672, and their difference is 1974. What are the numbers?

29. A man paid $1250 for a horse and carriage, the horse being valued at $190 more than the carriage. What was the value of each?

30. At a town election the whole number of votes cast for two candidates was 3789, and the majority for the successful candidate was 227. How many votes did each receive?

31. Two men are worth $28475, and one is worth $4625 more than the other. How much is each man worth?

32. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, using the same number of boxes of each kind. How many boxes will be required?

33. Sold a quantity of wood for $2492, that cost $1424, thus gaining $3 a cord. How many cords were there, and what was the cost per cord?

34. What number divided by 36, the quotient increased by 48, the sum diminished by 37, the remainder multiplied by 14, and the product increased by 216 ÷ 72, is 269?

Find the missing *term* in the following equations:

35. $(15341 \div 29) \times (8430 \div 1405) = 1587 \times ?$

36. $[4500 + (12000 - 1375) \div \overline{121 \times 25}] \times 48 = ? \quad \times 24 ?$

37. $732 \times 6 \div (\overline{15 \times 24} \div \overline{9 \times 10}) + (42 \times \overline{234 \div 26}) = ?$

38. $450 + \overline{(24 - 12) \times 5} \div (90 \div 6) + (\overline{3 \times 11} - 18) = ?$

146. Let the pupil be required to illustrate the following problems by original examples:

PROBLEM 1. Given several numbers, to find their sum.

2. Given the sum of several numbers and all of them but one, to find that one.

3. Given the parts, to find the whole.

4. Given the whole and all the parts but one, to find that one.

5. Given two numbers, to find their difference.

6. Given the greater of two numbers and their difference, to find the less.

7. Given the less of two numbers and their difference, to find the greater.

8. Given the minuend and subtrahend, to find the remainder.

9. Given the minuend and remainder, to find the subtrahend.

10. Given the subtrahend and remainder, to find the minuend.

11. Given two or more numbers, to find their product.

12. Given the product and one of two factors, to find the other factor.

13. Given the multiplicand and multiplier, to find the product.

14. Given the product and multiplicand, to find the multiplier.

15. Given the product and multiplier, to find the multiplicand.

16. Given two numbers, to find their quotient.

17. Given the divisor and dividend, to find the quotient.

18. Given the divisor and quotient, to find the dividend.

19. Given the dividend and quotient, to find the divisor.

20. Given the divisor, quotient, and remainder, to find the dividend.

21. Given the dividend, quotient, and remainder, to find the divisor.

22. Given the final quotient of a continued division and the several divisors, to find the dividend.

23. Given the quotient of a continued division, the first dividend, and all the divisors but one, to find that divisor.

24. Given the dividend and several divisors of a continued division, to find the quotient.

25. Given two or more sets of numbers, to find the difference of their sums.

26. Given two or more sets of factors, to find the sum of their products.

27. Given two or more sets of factors, to find the difference of their products.

28. Given the sum and the difference of two numbers, to find the numbers.

147. SYNOPSIS FOR REVIEW.

- DIVISION.
  - 1. **Definitions.**
    - 1. Division. 2. Dividend. 3. Divisor. 4. Quotient. 5. Remainder. 6. Sign of Division.
  - 2. **Principles**, 1, 2, and 3.
  - 3. **Relation of Division to Subtraction.**
  - 4. **Relation of Division to Multiplication.**
  - 5. **Objects of Division.**
    - 1. Illustrate.
    - 2. “
  - 6. **Equal Parts.**
    - 1, 2.
  - 7. **Short Division.**
    - 1. Definition.
    - 2. Method.
  - 8. **Long Division.**
    - 1. Definition.
    - 2. Method.
    - 3. Rule, I—VI.
    - 4. Proof.
  - 9. **Division of Dollars and Cents.**
    - 1. When divisor and dividend are concrete, but unlike.
    - 2. How to change dollars to cents.
    - 3. How to change dollars and cents to cents.
  - 10. **Division by Factors.**
    - 1. Method.
    - 2. Rule, I, II, III, IV.
  - 11. **When the Divisor has Ciphers on the Right.**
    - 1. Method.
    - 2. Rule, I, II, III.
  - 12. **General Principles of Division**, 1, 2, 3.
  - 13. **General Law.**

PROPERTIES OF NUMBERS

148. 1. What two numbers, besides the number itself and 1, will give a product of 8? 16? 25? 42? 64?

2. What numbers, other than the given number and 1, will exactly divide 9? 15? 36? 48? 55?

3. Of what sets of two numbers is 24 the product?

4. Of what sets of three numbers is 36 the product?

5. What are the smallest numbers, other than 1, that will exactly divide 18? 21? 49? 55?

6. What is the largest number, other than the given number itself, that will exactly divide 22? 24? 30? 40?

7. Name the numbers between 12 and 30, that are the product of *two* factors greater than 1. Between 30 and 50.

8. Name the numbers between 5 and 20, that have no other factors than the numbers themselves and 1.

9. Of what number are 7 and 8 the factors? 2, 5, and 7? 4, 5, and 3? 2, 3, 5, and 10?

DEFINITIONS.

149. The ***Properties of Numbers*** are those qualities or elements which necessarily belong to numbers.

All numbers are either *Integral* or *Fractional.*

150. An ***Integral Number,*** or ***Integer,*** is a number representing whole things. (**4.**)

Thus, 8, 23, 30 men, 45 pounds are integral numbers.

Integral numbers are either *Even* or *Odd, Prime* or *Composite.*

151. An ***Even Number*** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are *even*.

152. An ***Odd Number*** is a number that is not exactly divisible by 2.

All numbers whose unit figure is 1, 3, 5, 7, or 9, are *odd*.

153. A ***Prime Number*** is a number that has no integral factors except unity and itself.

Thus, 2, 3, 5, 11, 23, etc., are prime numbers.

154. A ***Composite Number*** is a number that has other *integral* factors besides unity and itself.

Thus, 21 is a composite number, since $21 = 7 \times 3$.

All *prime* numbers except two are *odd*.

155. The ***Factors*** of a number, are the numbers which multiplied together will produce it. (**108.**)

Thus, 7 and 8 are factors of 56; 3, 4, and 7, of 84.

156. A ***Prime Factor*** is a prime number used as a *factor*. (**153.**)

The *prime factors* of a number are also the *prime divisors* of it.

157. An ***Exact Divisor*** of a number is one that will divide that number without a remainder.

Thus, 6 is an exact divisor of 48, and 9 an exact divisor of 72.

1. The *Exact Divisors* of a number are also the *factors* of that number.

2. An exact divisor of a number is sometimes called the *measure* of that number.

3. When a number is a factor, or divisor, of each of *two* or *more* numbers, it is called a *common* factor, or divisor, of those numbers.

158. Numbers are ***prime to each other*** when they *have no common integral factors, or divisors.*

Thus, 9 and 14, 16 and 25 are prime to each other.

DIVISIBILITY OF NUMBERS.

159. A number is said to be divisible by another, when there is no remainder after dividing. Any number is *divisible*

1. By 2, if it is an *even* number.

Thus, 20, 24, 36, and 44 are divisible by 2.

2. By 3, if the sum of its digits is divisible by 3.

Thus, 135, 471, and 1134 are divisible by 3.

3. By 4, if its two right-hand figures are ciphers, or express a number divisible by 4.

Thus, 300, 432, and 1548 are divisible by 4.

4. By 5, if it ends with a cipher or 5.

Thus, 30, 45, and 235 are divisible by 5.

5. By 6, if it is an *even* number and divisible by 3.

Thus, 168, 402, and 1314 are divisible by 6.

6. By 8, if its three right-hand figures are ciphers, or express a number divisible by 8.

Thus, 3000, 2728, and 10576 are divisible by 8.

7. By 9, if the sum of its digits is divisible by 9.

Thus, 217683 and 401301 are divisible by 9.

8. By 10, if it ends with one or more ciphers.

Thus, 40, 500, 3000 are respectively divisible by 10, 100, and 1000.

9. By 7, 11, and 13, if it consists of but *four* places, the first and fourth being occupied by the same significant figures, and the second and third by ciphers.

Thus, 2002, 3003, and 5005, are divisible by 7, 11, and 13.

10. An *odd* number is not divisible by an *even* number.

11. If an *even* number is divisible by an *odd* number, the quotient will be an *even* number.

Thus, the quotient of 36 divided by 9, is 4; of 42 by 7, is 6.

12. If an even number is divisible by an odd number, it is also divisible by *twice* that number.

Thus, 28 is divisible by 7, and also by *twice* 7.

13. Every prime number except 2, increased or else diminished by 1, is divisible by 4.

Thus, 11 *increased* by 1, or 17 *diminished* by 1, is divisible by 4.

14. Every prime number except 2 and 5, increased or else diminished by 1, is divisible by 6.

Thus, 24 increased by 1, or 31 diminished by 1, is divisible by 6.

EXERCISES.

160. Find by inspection some of the exact divisors of the following numbers:

1. 1536.
2. 1683.
3. 3348.
4. 6105.
5. 12936.
6. 43560.
7. 32472.
8. 71460.
9. 197200.

FACTORING.

ORAL EXERCISES.

161. 1. What are the even numbers from 12 to 36?

2. What are the odd numbers from 12 to 36?

3. What are the prime numbers from 12 to 36?

4. What are the composite numbers from 12 to 36?

5. Name all the prime factors of 36. 2 - 3.

6. Name all the composite factors of 36.

7. What are the prime factors of 35? 49? 60?

8. What are the composite factors of 32? 48? 72?

9. What prime factor is common to 21 and 42?

10. What composite factors are common to 36 and 72?

11. What factors are common to 18 and 30? To their *sum* and *difference?*

12. What factors are common to the sum and difference of 20 and 40?

13. What prime factor is common to 14 and 4 times 14?

14. What two composite factors are common to 24 and 3 times 24?

15. What is the largest, and what the smallest prime factor of 13, 30, and 45?

DEFINITIONS AND PRINCIPLES.

162. ***Factoring*** is the resolving of a composite number into its factors, and is performed by division.

163. An ***Exponent*** is a small figure written at the right of a number, and a little above, to show how many times the number is to be used as a factor.

Thus, $2^3 = 2 \times 2 \times 2$, and denotes that 2 is used as a factor 3 *times.* $5^4 = 5 \times 5 \times 5 \times 5$, and denotes that 5 is used as a factor 4 *times.*

164. PRINCIPLES.—1. *The prime factors of a number, or the product of two or more of them, are the only exact divisors of that number.*

2. *A factor of a number is a factor also of any product of that number.*

3. *A factor common to two or more numbers is a factor also of their sum and difference.*

4. *Every composite number is equal to the product of its prime factors.*

WRITTEN EXERCISES.

165. To find all the prime factors of a composite number.

1. What are the prime factors of 2772?

OPERATION.

$$\begin{array}{r|l} 2 & 2772 \\ \hline 2 & 1386 \\ \hline 3 & 693 \\ \hline 3 & 231 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

ANALYSIS.—Since the given number is even, divide it by 2, the least prime factor, and the result also by 2, which gives an odd number for a quotient.

Next divide by the prime factors 3, 3, and 7, successively, obtaining for the last quotient 11, which not being divisible, is a prime factor of the given number. Hence the divisors 2, 2, 3, 3, 7, and the last quotient 11, are all the prime factors, or divisors, of 2772, and may be written 2^2, 3^2, 7, 11.

In like manner find the prime factors or divisors

2. Of 1050.
3. Of 1140.
4. Of 2445.
5. Of 2366.
6. Of 2205.
7. Of 2310.

RULE.—*Divide the given number by any prime factor of it, and the resulting quotient by another, and so continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

PROOF.—*The product of all the prime factors is equal to the given number.* (PRIN. 4.)

Resolve the following numbers into their prime factors:

8. 1155.
9. 2934.
10. 6300.
11. 2205.
12. 13981.
13. 32320.
14. 21504.
15. 29925.
16. 12673.
17. 10010.
18. 28665.
19. 31570.

COMMON DIVISORS.

ORAL EXERCISES.

166. 1. Name two exact divisors of 12. Of 15. Of 20.

2. Name three exact divisors of 24. Of 48. Of 72.

3. What number is an exact divisor of 27 and of 56?

4. What are the prime divisors of 15? 55? 49? 77?

5. What are the composite divisors of 72? 84? 120?

6. What prime divisor is common to 28, 35, and 42?

7. Name a common measure of 22, 44, and 66.

8. Name the greatest common measure of 16, 32, and 64.

9. Of what three numbers is 12 a common divisor?

10. What two numbers will exactly divide 15 and 30? Their *sum* and *difference?*

11. What is the smallest exact divisor of the sum and difference of 10 and 15? Of 21 and 56?

12. What is the greatest exact divisor of the sum and difference of 16 and 24? Of 18 and 45?

13. Find the greatest common measure of 14, 42, and 56.

14. Find the greatest common divisor of 27, 36, and 45.

DEFINITIONS AND PRINCIPLES.

167. A ***Common Divisor*** of two or more numbers is a *common factor* of each of them.

168. The ***Greatest Common Divisor*** of two or more numbers is the *greatest common factor,* and is the product of *all* the common *prime* factors, and no more.

169. PRINCIPLES.—1. *The only exact divisors of a number are its prime factors, or the product of two or more of them.*

2. *An exact divisor divides any number of times its dividend.*

3. *A common divisor of two or more numbers will divide their sum, and also their difference.*

4. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

WRITTEN EXERCISES.

170. When the numbers can be readily factored.

1. What is the greatest common divisor of 42, 63, and 126?

1ST OPERATION.

$$\begin{aligned} 42 &= 7 \times 3 \times 2 \\ 63 &= 7 \times 3 \times 3 \\ 126 &= 7 \times 3 \times 6 \end{aligned}$$

ANALYSIS.—By factoring the given numbers, the prime factors common to *all* of them are 7 and 3. Hence 7 × 3 = 21 is the greatest common divisor of 42, 63, and 126. (PRIN. 4.)

2D OPERATION.

$$\begin{array}{r|rrr} 3 & 42 & 63 & 126 \\ \hline 7 & 14 & 21 & 42 \\ \hline & 2 & 3 & 6 \end{array}$$

ANALYSIS.—Since the given numbers are exactly divisible by 3, and the resulting quotients by 7, they are also divisible by 7 × 3, or 21. (PRIN. 1.)

If there were other factors of the greatest common divisor, then the quotients 2, 3, and 6 would be exactly divisible by them.

Find the greatest common divisor

2. Of 42 and 112.
3. Of 96 and 544.
4. Of 40, 75, and 100.
5. Of 72, 126, and 216.

RULE.—*Separate the numbers into their prime factors and find the product of all that are common.* Or,

I. *Write the numbers in a line, and divide by any prime factor common to all the numbers.*

II. *Divide the quotients in like manner, and so continue the division till all the quotients are prime to each other.*

III. *The product of all the divisors will be the greatest common divisor.* (PRIN. 4.)

What is the greatest common divisor

6. Of 144 and 720?

7. Of 308 and 506?

8. Of 126, 210, and 252?

9. Of 72, 96, 120, and 384?

171. When the numbers cannot be readily factored.

1. Find the greatest common divisor of 527 and 1207.

OPERATION.

		1207
527	2	1054
459	3	153
68	2	136
68	4	17

ANALYSIS.—Draw two vertical lines, and place the larger number on the right, and the smaller on the left, one line lower down. Divide 1207 by 527, and write the quotient 2 between the vertical lines, the product, 1054, under the greater number, and the remainder 153, below.

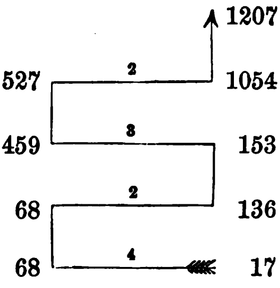
Next, divide 527 by this remainder 153, writing the quotient 3 between the verticals, the product 459, on the left, and the remainder 68, below.

Again, divide the last divisor 153, by 68, and write the product, and remainder in the same order as before.

Finally, dividing the last divisor 68, by the last remainder 17, there is no remainder. Hence 17, the last divisor, is the greatest common divisor of 537 and 1207.

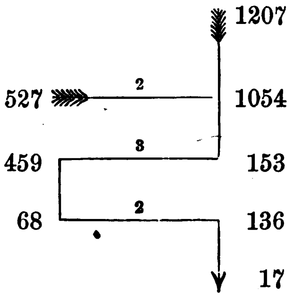
PROOF.—Now, observing that the *dividend* is always the *sum* of the product and remainder, and that the *remainder* is always the *difference* of the dividend and product, trace the work in the reverse order, as indicated by the arrow line in the diagram below.

ILLUSTRATION.



17 divides 68, as proved by the last division; it will also divide 2 times 68, or 136 (PRIN. 2). Since 17 divides both itself and 136, it will divide 153, their sum (PRIN. 3). It will also divide 3 times 153, or 459 (PRIN. 2); and since it is a common divisor of 459 and 68, it must divide their *sum*, 527, which is one of the given numbers. It will also divide 2 times 527, or 1054 (PRIN. 2); and since it divides 1054 and 153, it must divide their *sum*, 1207, the greater number (PRIN. 3.) Hence, 17 is a *common divisor* of the given numbers.

Again, tracing the work in the direct order, as indicated in the following diagram, the *greatest* common divisor, *whatever it is,* must divide 2 times 527, or 1054 (Prin. 2). And since it will divide both 1054 and 1207, it must divide their difference, 153 (Prin. 3). It will also divide 3 times 153, or 459 (Prin. 2); and as it will divide both 459 and 527, it must divide their *difference,* 68 (Prin. 3). It will also divide 2 times 68, or 136 (Prin. 2); and as it will divide both 136 and 153, it must divide their *difference,* 17 (Prin. 3); *hence, it cannot be greater than* 17.



Thus it has been shown,

1st. That 17 is a *common divisor* of the given numbers.

2d. That their greatest common divisor, whatever it be, cannot be *greater than* 17. Hence it must be 17.

In like manner find the greatest common divisor

2. Of 316 and 664.
3. Of 679 and 1869.
4. Of 1080 and 189.
5. Of 2192 and 458.
6. Of 825 and 1372.
7. Of 2041 and 8476.
8. Of 7241 and 10907.
9. Of 2373 and 6667.

Rule.—I. *Draw two vertical lines, and write the two numbers, one on each side, the greater number one line above the less.*

II. *Divide the greater number by the less, writing the quotient between the verticals, the product under the dividend, and the remainder below.*

III. *Divide the less number by the remainder, the last divisor by the last remainder, and so on, till nothing remains. The last divisor is the greatest common divisor sought.*

IV. *If more than two numbers are given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last; the last common divisor found is the greatest common divisor of all the given numbers.*

10. What is the greatest number that will divide 3281 and 10778? 10353 and 14877?

11. What is the greatest number that will divide 620, 1116, and 1488? 396, 5184, and 6914?

12. A man having a piece of land, the sides of which are 240 feet, 648 feet, and 420 feet, wishes to inclose it with a fence having pannels of the greatest possible uniform length; what will be the length of each pannel?

13. A farmer wishes to put 231 bushels of corn, 393 bushels of wheat, and 609 bushels of oats into the largest bags of equal size, that will exactly hold each kind. How many bushels must each bag hold?

14. A forwarding merchant has 15292 bushels of wheat, 1520 bushels of corn, and 504 bushels of beans, which he wishes to ship, in the fewest bags of equal size that will exactly hold either kind of grain; how many bags will it take?

15. Three persons have respectively $630, $1134, and $1386, with which they agree to purchase horses, at the highest price per head, that will allow each man to invest all his money. How many horses can each man buy?

16. How many rails will inclose a field 5850 feet long by 1729 feet wide, the fence being straight, and 7 rails high, and the rails of equal length, and the longest that can be used?

MULTIPLES.

ORAL EXERCISES.

172. 1. What numbers between 5 and 30 are exactly divisible by 4? By 6? 7? 8? 9?

2. What numbers less than 40 are exactly divisible by 7?

3. What prime factors are common to 6, and 5 times 6?

4. Name some numbers exactly divisible by 4 and 6; by 3 and 7; by 5 and 7; by 8 and 10.

5. By what three prime numbers can 42 be divided?

6. Name some numbers of which 3 and 4 are factors.

7. Find the least number exactly divisible by 3, 4, and 5.

DEFINITIONS AND PRINCIPLES.

173. A ***Multiple*** of a number is a number exactly divisible by the given number; or, it is any product or dividend of which a given number is a *factor.*

1. A number may have an *unlimited number* of multiples.

2. A number is a *divisor* of all its multiples and a *multiple* of all its divisors.

174. A ***Common Multiple*** of two or more given numbers is a number exactly divisible by each of them.

175. The ***Least Common Multiple*** of two or more given numbers is the least number exactly divisible by each of them.

Two or more numbers can have but *one* least common multiple.

176. Principles.—1. *A multiple of a number contains all the prime factors of that number.*

2. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.* Hence,

3. *The least common multiple of two or more numbers is the least number that contains all the prime factors of each of those numbers.*

4. *A common multiple of two or more numbers may be found by multiplying the given numbers together.*

WRITTEN EXERCISES.

177. To find the least common multiple.

FIRST METHOD.

1. Find the least common multiple of 30, 42, and 66.

OPERATION.

$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$
$$66 = 2 \times 3 \times 11$$
$$2 \times 3 \times 11 \times 7 \times 5 = 2310$$

ANALYSIS.—The least common multiple cannot be less than the largest number 66, since it must contain 66; hence it must contain all the prime factors of 66, 2, 3, and 11. (PRIN. 1.) But the least common multiple of 66 must also contain all the prime factors of each of the other numbers, and since the prime factors 2 and 3 of 66 are common also to 42 and 30 omit them, and annex the factors 7 and 5 to those of 66, and the series 2, 3, 11, 7, and 5 are all the prime factors of the given numbers, and their product 2 × 3 × 11 × 7 × 5 = 2310, is the least common multiple of the given numbers. (PRIN. 3.)

2. Find the least common multiple of 24, 42, and 17.

3. Find the least common multiple of 8, 12, 20, and 30.

4. Find the least common multiple of 10, 45, 75, and 90.

RULE.—I. *Resolve each of the given numbers into its prime factors.*

II. *Multiply together all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

Find the least common multiple

5. Of 30, 66, 78, and 42.
6. Of 21, 30, 44, and 126.
7. Of 16, 60, 140, and 210.
8. Of 16, 48, 80, 32, and 66.

SECOND METHOD.

178. 1. Find the least common multiple of 18, 24, and 54.

OPERATION.

2	18	24	54
3	9	12	27
3	3	4	9
		4	3

ANALYSIS.—Write the numbers in a horizontal line, with a vertical line at the left.

Since 2 is a prime factor of one or more of the given numbers, it must also be a factor of the least common multiple of those numbers. (PRIN. 3.) Hence, divide by 2 and write the quotients underneath. For a like reason divide again successively by 3 and 3, writing the quotients and undivided numbers in a line below, omitting to write any quotient when it is 1.

Since there is no factor common to 4 and 3, they are prime to each other, and hence the divisors 2, 3, and 3, with the numbers 4 and 3 in the last line, are all the prime factors of the given numbers, and their product 216 is the least common multiple. (PRIN. 3.)

If in any example, any of the smaller numbers are exactly contained in the larger, they may be omitted in finding the least common multiple, inasmuch as a number that will contain a given number, will contain any factor of that number.

Thus, if required to find the least common multiple of 8, 12, 24, 72, and 120, omit all the numbers except 72 and 120, since the others are factors of these, and the least common multiple of 72 and 120, will be the least common multiple of *all* the numbers.

2. Find the least common multiple of 32, 34, and 36. ✓
3. Find the least common multiple of 84, 100, and 224.

RULE.—I. *Write the numbers in a horizontal line, omitting such of the smaller numbers as are factors of the larger, and draw a vertical line at the left.*

II. *Divide by any prime factor that will exactly divide two or more of the given numbers, and write the quotients and undivided numbers in a line underneath.*

III. *In like manner divide the quotients and undivided numbers until they are prime to each other.*

IV. *The product of the divisors and the final quotients and undivided numbers, is the least common multiple.*

What is the least common multiple

4. Of 4662, and 5698?
5. Of 312, 260, and 390?
6. Of 24, 10, 32, 45 and 25?
7. Of 153, 204, 102, and 1020?

8. Find the least common multiple of the first eight even numbers.

9. Find the least common multiple of the first five odd numbers.

10. What is the smallest number of oranges that can be equally distributed among 16, 20, 24, or 30 boys?

11. What is the shortest piece of rope that can be cut exactly into pieces either 15, 18, or 20 feet long?

12. What is the smallest sum of money for which I could purchase an exact number of books, at $5, or $3, or $4, or $6 each?

13. What is the product of the least common multiple of 12, 16, 24, and 32, multiplied by their greatest common divisor?

14. Divide the least common multiple of 7, 42, 6, 9, 10, and 630, by the greatest common divisor of 110, 140, and 680.

15. What is the smallest sum of money, for which I can purchase an exact number of sheep at $8, or cows at $28, or oxen at $54, or horses at $162 each?

16. What is the smallest quantity of grain that will fill an exact number of bins, whether they hold 36, 48, 80, or 144 bushels?

CANCELLATION.

ORAL EXERCISES.

179. 1. Divide 72 by 24. One-half of 72 by one-half of 24. One-third of 72 by one-third of 24.

2. Divide one-fourth of 72 by one-eighth of 24.

3. Divide 36 by 9. One-third of 36 by one-third of 9.

4. What factors are common to 72 and 24?

5. What is the quotient of 12 × 6 divided by 12 × 2?

6. Divide $3 \times 3 \times 3$ by 3×3. $4 \times 5 \times 2$ by $2 \times 2 \times 5$.

7. Divide $7 \times 6 \times 2$ by $2 \times 6 \times 7$. $5 \times 6 \times 4$ by $3 \times 4 \times 6$.

DEFINITIONS AND PRINCIPLES.

180. ***Cancellation*** is the process of abridging operations in division by rejecting equal factors from both dividend and divisor.

181. PRINCIPLES.—1. *Rejecting a factor from any number divides the number by that factor.*

2. *Rejecting equal factors from both dividend and divisor does not change the quotient.*

WRITTEN EXERCISES.

182. 1. Divide 56×24, by 4×7.

1ST OPERATION.

$$\frac{56 \times 24}{48 \times 7} = \frac{\cancel{8} \times \cancel{7} \times \cancel{6} \times 4}{\cancel{8} \times \cancel{6} \times \cancel{7}} = 4$$

ANALYSIS.—Indicate the operation to be performed in the example, by writing the numbers that constitute the dividend, above a line, and those that constitute the divisor, below it.

Resolve these numbers into their factors, and the dividend will consist of $8 \times 7 \times 6 \times 4$, and the divisor of $8 \times 6 \times 7$. Rejecting equal *factors from* both dividend and divisor, there remains the factor 4 *in the dividend.* Hence the quotient is 4.

2D OPERATION.

$$\frac{\overset{\cancel{7}}{\cancel{5}\cancel{6}} \times \overset{4}{\cancel{2}\cancel{4}}}{\underset{\cancel{6}}{\cancel{4}\cancel{8}} \times \cancel{7}} = 4$$

ANALYSIS.—Since it is evident that 8 will divide both 56 and 48, reject 8 as a factor of 56, retaining the factor 7, and also of 48, retaining the factor 6.

Again, since 6 will divide both 24 in the dividend and 6 in the divisor, reject 6 as a factor from both, retaining the factor 4 in the dividend. Finally, rejecting the factor 7, common both to the dividend and to the divisor, there remains only the factor 4 in the dividend, which is the required quotient.

2. Divide the product of 44, 30, 7, and 6, by the product of 33, 18, and 14; or, divide 55440 by 8316.

OPERATION.

$$\frac{55440}{8316} = \frac{\overset{2}{\cancel{4}\cancel{4}} \times \overset{10}{\cancel{3}\cancel{0}} \times \cancel{7} \times \cancel{6}}{\underset{3}{\cancel{3}\cancel{3}} \times \underset{\cancel{3}}{\cancel{1}\cancel{8}} \times \underset{\cancel{2}}{\cancel{1}\cancel{4}}} = \frac{10 \times 2}{3} = 6\tfrac{2}{3}$$

Or,

$$\begin{array}{r|l} 3\ \cancel{3}\cancel{3} & \cancel{4}\cancel{4}\ 2 \\ \cancel{3}\ \cancel{1}\cancel{8} & \cancel{3}\cancel{0}\ 10 \\ \cancel{2}\ \cancel{1}\cancel{4} & \cancel{7} \\ & \cancel{6} \\ \hline 3 & 20 \\ \cline{2-2} & 6\tfrac{2}{3} \end{array}$$

By many it is thought more convenient to write the factors of the *dividend* on the *right* of a *vertical* line, and the factors of the *divisor* on the *left.*

3. Divide 13 × 7 × 5 × 3 by 3 × 5 × 7.
4. Divide 42 × 18 × 6 × 4 by 36 × 21 × 6.

RULE.—I. *Cancel all the factors common to both dividend and divisor.*

II. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

When a factor equal to the number itself is canceled, the unit 1 remains, since a number divided by itself gives a quotient of 1. If the 1 occur in the dividend, it must be retained; if in the divisor, *it need not be* regarded.

5. What is the quotient of 35 × 33 × 28, divided by 15 × 14 × 11?

6. What is the quotient of 140 × 39 × 13 × 7, divided by 7 × 26 × 21?

7. Multiply 11 times 21 by 26, and divide the product by 14 times 13.

8. How many times is the continued product of 14, 9, 3, 20, 5, and 6 contained in the continued product of 183, 18, 70, 12, and 5?

9. If 213 × 190 × 84 × 264 is the dividend, and 56 times 36 multiplied by 30 is the divisor, what is the quotient?

10. $(240 \times 56 \times 18) \div (60 \times 28 \times 9) = ?$

11. $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6) = ?$

12. $(66 \times 18 \times 27 \times 25) \div (84 \times 45 \times 7 \times 30) = ?$

13. $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20) = ?$

14. Multiply 64 by 7 times 31, divide the product by 8 times 56, multiply this quotient by 15 times 88, divide the product by 55, multiply this quotient by 13, and divide the product by 4 times 6. What is the quotient?

15. Find the quotient of $\frac{12 \times 60 \times 27 \times 35}{7 \times 15 \times 42 \times 108}$.

16. Find the quotient of $\frac{77 \times 100 \times 18 \times 64}{25 \times 11 \times 49 \times 16}$.

17. How many tons of hay at $18, must be given for 45 cords of wood at $4 a cord?

18. How many flour barrels at $.80 each, will pay for 112 bushels of corn at $.70 a bushel?

19. How many tubs of butter, each containing 56 pounds, at 30 cents a pound, must be given for 7 barrels of sugar, each containing 195 pounds, at 10 cents a pound?

20. A laborer gave 12 days' work for 48 bushels of potatoes, worth 4 shillings a bushel. What were his daily earnings?

21. A grocer sold 24 boxes of soap, each containing 55 pounds, at 10 cents a pound, and received as pay 88 barrels of apples, each containing 3 bushels. How much were the apples worth a bushel?

22. Sold 20 pounds of butter at 27 cents a pound, which exactly paid for 15 pounds of coffee. What was the price of the coffee a pound?

23. A farmer exchanged 240 bushels of corn worth $.75 a bushel, for an equal number of bushels of barley worth $1 a bushel, and oats worth $.50 a bushel. How many bushels of each did he receive?

24. A farmer bought two kinds of cloth, one kind at $.75 a yard, and the other at $.90, buying twice as many yards of the first kind as of the second. He paid for the same 132 pounds of butter at 40 cents a pound. How many yards of each kind of cloth did he buy?

25. A merchant bought 6 loads of oats, each load containing 22 bags, and each bag 2 bushels, worth 56 cents a bushel. He gave in payment 8 boxes of tea, each containing 24 pounds. What was the tea worth a pound?

26. How many bushels of oats at $.60 a bushel will pay for 12 tons of coal at $7.20 a ton?

27. How many chests of tea, each containing 63 pounds, worth 7 shillings a pound, must be given for 21 bags of coffee, each weighing 28 pounds, worth 3 shillings a pound?

28. How many days' work, at $1.25 a day will pay for 75 bushels of corn, at $.80 a bushel?

183. SYNOPSIS FOR REVIEW.

PROPERTIES OF NUMBERS.

- 1. DEFINITIONS.
  - 1. Properties of Numbers. 2. Integral Number, or Integer. 3. Even Number. 4. Odd Number. 5. Prime Number. 6. Composite Number. 7. Factors. 8. Prime Factor. 9. Exact Divisor.
- 2. DIVISIBILITY OF NUMBERS.
  - 1. How to find whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10; also by 7, 11, or 13.
  - 2. Other properties of even and of prime numbers.
- 3. FACTORING.
  - 1. Definitions.
    - 1. *Factoring.*
    - 2. *Exponent.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. Rule.
  - 4. Proof.
- 4. COMMON DIVISORS.
  - 1. Definitions.
    - 1. *Divisor.*
    - 2. *Common Divisor.*
    - 3. *G. C. Divisor.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. Rule (1st), I, II, III.
  - 4. Rule (2d), I, II, III, IV.
- 5. MULTIPLES.
  - 1. Definitions.
    - 1. *Multiple.*
    - 2. *C. Multiple.*
    - 3. *L. C. Multiple.*
  - 2. Principles, 1, 2, 3.
  - 3. Rule (1st), I, II.
  - 4. Rule (2d), I, II, III, IV.
- 6. CANCELLATION.
  - 1. Definition.
  - 2. Principles, 1, 2.
  - 3. Rule, I, II.

FRACTIONS

ORAL EXERCISES.

184. 1. If any unit, as an apple, or a yard, be divided into 2 equal parts, what is each part named? *One-half.*

2. If the unit be divided into 3 equal parts, what name is given to 1 of the parts? To 2 of the parts?

3. If the unit be divided into 5 equal parts, what is each part named? What name is given to 3 of the parts?

4. How many *halves* are there in a unit? How many *thirds? Fourths? Fifths? Sixths? Sevenths?*

5. If a mile be divided into 4 equal parts, what part of the whole mile is 1 of the parts? 3 of the parts?

6. What is 1 of 5 equal parts of a unit called? What are 2 of 6 equal parts called? 4 of 10 equal parts?

7. What is meant by 1 sixth of a unit? By 3 fourths?

8. What are 3 of the 7 equal parts of a week called?

9. Which is the *smaller,* one-third or one-fourth? One-fifth or one-third?

10. Which is the *greater,* one-fourth or one-sixth?

185. PRINCIPLES.—1. *The LESS the NUMBER of equal parts into which a unit is divided, the GREATER is the VALUE of each part.*

2. *The GREATER the NUMBER of equal parts into which a unit is divided, the LESS is the VALUE of each part.*

DEFINITIONS.

186. A ***Fraction*** is one or more of the equal parts of a unit. Thus, 1 *half* and 2 *thirds* are fractions.

187. A ***Fractional Unit*** is *one* of the equal parts into which any integral unit is divided. Thus, 1 fourth and 1 fifth are fractional units of *fourths* and *fifths*.

Fractional units take their *name* and their *value* from the *number* of parts into which the *integral* unit is divided.

188. A *fraction* is usually expressed by two numbers, called the *Numerator* and *Denominator*, one written over the other with a line between them. When written in this form they are sometimes called *Common Fractions.* Thus,

One-third	is written	$\frac{1}{3}$	Nine-tenths	is written	$\frac{9}{10}$
Three-fourths	“	$\frac{3}{4}$	Seven-twentieths	“	$\frac{7}{20}$
Five-sixths	“	$\frac{5}{6}$	Twelve-thirty-fifths	“	$\frac{12}{35}$
Seven-eighths	“	$\frac{7}{8}$	Thirty-six forty-ninths	“	$\frac{36}{49}$

189. The ***Denominator*** of a fraction shows the number of *equal parts* into which the unit is divided, and also indicates the *name* of these parts. It is written *below* the line.

Thus, in the fraction $\frac{7}{8}$, 8 is the *denominator* and shows that the unit is divided into *eight* equal parts, named *eighths.*

190. The ***Numerator*** of a fraction shows the *number* of equal parts taken to form the fraction. It is written *above* the line.

Thus, in $\frac{7}{8}$, 7 is the *numerator,* and shows that 7 of the 8 equal parts are taken, or expressed by the fraction.

191. The ***Terms*** of a fraction are its numerator and denominator. Thus, 6 and 7 are the *terms* of the fraction $\frac{6}{7}$.

Express by figures,

1. Five-ninths.
2. Seven twenty-fifths.
3. Nine-eighteenths.
4. Twelve twentieths.
5. Eight thirty-sixths.
6. Twenty-six forty-eighths.
7. Twenty-seven two-hundredths.
8. Forty-three ninety-ninths.
9. Sixteen one-hundred-eighths.
10. Fifty-five eighty-ninths.

Copy and read,

7. $\frac{7}{11}$; $\frac{9}{13}$; $\frac{11}{27}$; $\frac{16}{73}$; $\frac{27}{112}$; $\frac{64}{148}$; $\frac{95}{208}$.

8. $\frac{14}{116}$; $\frac{9}{84}$; $\frac{23}{180}$; $\frac{120}{450}$; $\frac{225}{876}$; $\frac{32}{960}$; $\frac{204}{600}$.

192. *Fractions* are *Proper* or *Improper*.

193. A ***Proper Fraction*** is a fraction whose numerator is less than its denominator. Its *value* is less than a unit. Thus, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{11}{14}$ are proper fractions.

194. An ***Improper Fraction*** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to, or greater than a unit. Thus, $\frac{6}{6}$, $\frac{10}{4}$, and $\frac{21}{9}$ are improper fractions.

195. A ***Mixed Number*** is an integer and a fraction united. Thus, $12\frac{5}{8}$ is equivalent to $12 + \frac{5}{8}$.

196. The ***Reciprocal*** of a number is 1 divided by that number. Thus, the reciprocal of 9 is $1 \div 9 = \frac{1}{9}$; of 16, it is $1 \div 16 = \frac{1}{16}$, etc.

197. The ***Reciprocal of a Fraction*** is 1 divided by that fraction, or it is the fraction *inverted*. Thus, the reciprocal of $\frac{3}{4}$ is $1 \div \frac{3}{4} = \frac{4}{3}$; of $\frac{7}{12}$, it is $\frac{12}{7}$.

198. The ***Value*** of a fraction is the quotient of its numerator divided by its denominator. Thus, $\frac{12}{3} = 4$.

1. Analyze the fraction $\frac{7}{8}$.

ANALYSIS.—$\frac{7}{8}$ is a fraction; 8 is the *denominator*, and shows that the unit is divided into 8 equal parts; $\frac{1}{8}$ is the *fractional unit*, since it is *one* of the *eight* equal parts into which the unit is divided; 7 is the *numerator*, and shows that *seven* of these equal parts are taken; 7 and 8 are the *terms* of the fraction. It is a *proper* fraction, since the numerator is less than the denominator; its *value* is *less* than 1; and it is read *seven-eighths*.

In like manner, analyze

2. $\frac{8}{9}$. | 3. $\frac{11}{12}$. | 4. $\frac{9}{4}$. | 5. $\frac{21}{28}$. | 6. $\frac{17}{9}$.

199. Since fractions indicate division, all changes in the *terms* of a fraction will affect the *value* of the fraction according to the laws of division; hence if we substitute the General Principles of Division (**142**), we shall have the following

200. GENERAL PRINCIPLES OF FRACTIONS.

1. *Multiplying the numerator*, or *Dividing the denominator,*	*Multiplies the fraction.*
2. *Dividing the numerator*, or *Multiplying the denominator,*	*Divides the fraction.*
3. *Multiplying or dividing both numerator and denominator by the same number,*	*Does not change the value of the fraction.*

201. These *three principles* may be embraced in one

GENERAL LAW.

A change in the NUMERATOR *produces a* LIKE *change in the value of the fraction; but a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

REDUCTION.

202. To reduce fractions to higher or lower terms.

ORAL EXERCISES.

1. One-half is equal to how many fourths?

ANALYSIS.—Since 1 is equal to 4 *fourths*, $\frac{1}{2}$ is equal to 1 half of 4 fourths or 2 fourths. Hence $\frac{1}{2}$ is equal to $\frac{2}{4}$.

2. One-third of a mile is how many *sixths* of a mile?

3. One-half of a dollar is how many *fourths* of a dollar?

4. Name some equivalent fractions for halves. Thirds.

5. Express $\frac{2}{3}$ in terms 3 times as large. 4 times as large?

6. The denominators fourths, sixths, eighths, and tenths, are multiples of what number?

7. Multiply both terms of $\frac{3}{4}$ by 3, and show that the *value* of the fraction is not changed.

ANALYSIS.—If both terms of $\frac{3}{4}$ are multiplied by 3, the resulting fraction is $\frac{9}{12}$, which is equivalent to $\frac{3}{4}$, since the *fractional unit* is $\frac{1}{3}$ as great, while the number taken is 3 times as great.

8. Name three equivalent fractions for $\frac{4}{5}$; for $\frac{1}{6}$; for $\frac{3}{8}$.

9. Change $\frac{5}{6}$ to twelfths. To eighteenths.

10. 8 *twelfths* are how many *thirds?*

ANALYSIS.—Since 1 third is equal to 4 twelfths, 8 twelfths are equal to as many thirds as 4 twelfths are contained times in 8 twelfths, which is 2 times. Hence there are $\frac{2}{3}$ in $\frac{8}{12}$.

11. How many *fourths* of a rod are 9 *twelfths* of a rod?

12. Divide both terms of $\frac{15}{20}$ by 5, and show that the *value* of the fraction is not changed.

ANALYSIS.—If both terms of $\frac{15}{20}$ are divided by 5, the resulting fraction is $\frac{3}{4}$, which is equivalent to $\frac{15}{20}$, since the *fractional unit is 5 times as great*, while the number taken is $\frac{1}{5}$ as great.

13. Change $\frac{18}{36}$ to an equivalent fraction having a denominator 1 *half* as large. 1 *third* as large.

14. Change $\frac{9}{27}$ to a fraction having lower terms. $\frac{14}{21}$. $\frac{25}{30}$.

15. In what lower terms can $\frac{12}{24}$ be expressed?

16. Change $\frac{9}{36}$ to its lowest terms. $\frac{16}{20}$. $\frac{32}{40}$. $\frac{14}{35}$. $\frac{20}{25}$.

17. Name two common divisors of $\frac{18}{36}$. $\frac{24}{56}$. $\frac{30}{45}$. $\frac{33}{66}$.

18. Express $\frac{8}{16}$ in parts 4 times as great in *value*.

19. Express $\frac{12}{18}$ in parts 6 times as great in *value*.

DEFINITIONS.

203. ***Reduction of Fractions*** is the process of changing their *form* without altering their *value*.

204. A fraction is reduced to ***Higher Terms*** when the numerator and denominator are expressed in larger numbers. Thus, $\frac{3}{4} = \frac{6}{8}$, or $\frac{9}{12}$.

205. A fraction is reduced to ***Lower Terms*** when the numerator and denominator are expressed in smaller numbers. Thus, $\frac{8}{12} = \frac{4}{6}$, or $\frac{2}{3}$.

206. A fraction is reduced to its ***Lowest Terms*** when its numerator and denominator are *prime to each other*. Thus, $\frac{4}{10} = \frac{2}{5}$; $\frac{12}{16} = \frac{3}{4}$.

207. Fractions are changed to *higher terms* by Multiplication, and to *lower terms* by Division.

All higher terms of a fraction are multiples of its lowest terms.

208. Principle.—*Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.* (**200,** 3.)

WRITTEN EXERCISES.

209. 1. Change $\frac{5}{6}$ to a fraction whose denominator is 30.

OPERATION.

$30 \div 6 = 5$

$\frac{5\times5}{6\times5}=\frac{25}{30}$

ANALYSIS.—First, divide 30, the required denominator, by 6 the denominator of the given fraction. The quotient 5 is the factor employed to produce the required denominator. Hence, multiply both terms of $\frac{5}{6}$ by 5 (**200,** 3), and $\frac{25}{30}$ is the required fraction.

2. Change $\frac{7}{12}$ to a fraction, whose denominator is 96.
3. Change $\frac{11}{15}$ to a fraction, whose denominator is 105.
4. Reduce $\frac{72}{120}$ to its lowest terms.

OPERATION.

$\frac{72\div8}{120\div8}=\frac{9}{15}$; $\frac{9\div3}{15\div3}=\frac{3}{5}$

Or, $\frac{72\div24}{120\div24}=\frac{3}{5}$

ANALYSIS.—Dividing both terms of the given fraction $\frac{72}{120}$, by 8, (**200,** 3) the result is $\frac{9}{15}$. Again, dividing both terms of $\frac{9}{15}$ by 3, the result is $\frac{3}{5}$. Since the terms of $\frac{3}{5}$ are *prime to each other*, the lowest terms of $\frac{72}{120}$ are $\frac{3}{5}$.

The same result is obtained more directly, by dividing both terms by their greatest common divisor, 24.

5. Reduce $\frac{84}{96}$ to its lowest terms.
6. Reduce $\frac{75}{135}$ to its lowest terms.
7. Reduce $\frac{156}{208}$ to its lowest terms.

RULES.—1. To reduce a fraction to *higher terms.*

Divide the required denominator by the denominator of the given fraction, and multiply the terms of the given fraction by the quotient.

2.—To reduce a fraction to its *lowest terms.*

Reject all factors common to the terms of the given fraction. Or,

Divide the terms of the given fraction by their greatest common divisor.

8. Change $\frac{7}{15}$ to a fraction whose denominator is 180.

9. Reduce $\frac{7}{9}$ and $\frac{5}{21}$ each to sixty-thirds.

10. Reduce $\frac{5}{8}$, $\frac{1}{4}$, and $\frac{7}{10}$, each to 120ths.

11. Reduce $\frac{10}{11}$, $\frac{1}{6}$, $\frac{10}{22}$, and $\frac{21}{33}$, each to 132ds.

12. Change 168 ÷ 252 to the form of a fraction in its lowest terms. 81 ÷ 63. 160 ÷ 400. 324 ÷ 612.

Reduce to their lowest terms,

13. $\frac{288}{360}$.
14. $\frac{258}{282}$.
15. $\frac{288}{504}$.
16. $\frac{441}{462}$.
17. $\frac{639}{1737}$.
18. $\frac{172}{1118}$.
19. $\frac{568}{1544}$.
20. $\frac{1344}{1536}$.
21. $\frac{5643}{5940}$.
22. $\frac{4680}{10600}$.
23. $\frac{17696}{26640}$.
24. $\frac{4692}{161976}$.

210. To reduce an integer or a mixed number to an improper fraction.

ORAL EXERCISES.

1. In 3 units, how many fourths ?

ANALYSIS.—Since in 1 unit there are 4 *fourths*, in 3 units there are 3 times 4 fourths, or 12 fourths. Hence $3 = \frac{12}{4}$.

2. In 4 bushels, how many *eighths* of a bushel ?

3. How many *sevenths* of a week in 6 weeks ?

4. How many 9ths in 5 ? 6 ? 8 ? 10 ? 12 ?

5. How many tenths of a dollar in $7 ? In $9 ?

6. How many *half* dollars will pay for a ton of coal that cost $7 ? For a barrel of flour that cost $10 ?

7. How may an integer be changed to *thirds ?* To *sixths ?* To *eighths ?* To *tenths ?*

8. In $5\frac{3}{8}$ how many *eighths ?*

ANALYSIS.—Since 1 is equal to 8 *eighths*, 5 equals 5 times 8 eighths, or 40 eighths, and $\frac{3}{8}$ added make 43 eighths. Hence $5\frac{3}{8} = \frac{43}{8}$.

9. In $6\frac{3}{4}$ *cords of* wood, how many *fourths* of a cord ?

10. How many 6ths in \$$8\frac{5}{6}$? In $12\frac{1}{6}$ rods?

11. Among how many boys can you distribute $5\frac{3}{4}$ quarts of chestnuts, if you give $\frac{1}{4}$ of a quart to each?

12. Among how many poor families can $4\frac{5}{6}$ tons of coal be distributed, if each family receive $\frac{1}{6}$ of a ton?

WRITTEN EXERCISES.

211. 1. Change 75 to the form of a fraction having 27 for its denominator.

OPERATION.

$$75 \times 27 = 2025$$

$$75 = \frac{2025}{27}$$

ANALYSIS.—Since 1 is equal to 27 *twenty-sevenths*, 75 are equal to 75 times 27 twenty-sevenths, or 2025 twenty-sevenths. Hence $75 = \frac{2025}{27}$.

2. Change $49\frac{7}{12}$ to *twelfths.*

OPERATION.

$$\begin{array}{l} 49\frac{7}{12} \\ \underline{12} \\ 588 \text{ twelfths} \end{array}$$

$$\frac{588}{12} + \frac{7}{12} = \frac{595}{12}$$

ANALYSIS.—Since 1 is equal to 12 *twelfths*, 49 are equal to 49 times 12 twelfths, or 588 twelfths; to which add $\frac{7}{12}$, and the result is 595 twelfths. Hence $49\frac{7}{12} = \frac{595}{12}$.

An *integer* is reduced to a fractional *form* by writing 1 under it for a denominator. Thus, $9 = \frac{9}{1}$; $23 = \frac{23}{1}$.

3. Change 81 to a fraction having 24 for its denominator.

4. In 78 pounds, how many sixteenths of a pound?

5. In $42\frac{6}{7}$ weeks, how many sevenths of a week?

6. How many 20ths of a ton in $16\frac{17}{20}$ tons? In $21\frac{13}{20}$ tons?

RULE.—*Multiply the integer by the required denominator, and to the product, add the numerator of the fraction, and under the result write the required denominator.*

Reduce

7. 207 to fifteenths.
8. $136\frac{11}{18}$ to eighteenths.
9. $472\frac{7}{26}$ to twenty-sixths.
10. $543\frac{9}{40}$ to fortieths.
11. $184\frac{32}{95}$ to ninety-fifths.
12. $2014\frac{61}{84}$ to eighty-fourths.

13. Reduce $204\frac{11}{24}$ days to twenty-fourths of a day.
14. Change 312 to a fraction whose denominator is 126.
15. Reduce $2146\frac{3}{64}$ to an improper fraction.
16. Change $1006\frac{17}{90}$ to an improper fraction.

212. To reduce an improper fraction to an integer, or a mixed number.

ORAL EXERCISES.

1. How many units are $\frac{18}{4}$?

Analysis.—Since 4 *fourths* equal 1, 18 fourths are as many times 1 as 4 fourths are contained times in 18 fourths, which is $4\frac{2}{4}$ times.

2. How many times 1 are $\frac{28}{7}$? $\frac{46}{8}$? $\frac{75}{12}$? $\frac{60}{9}$? $\frac{90}{15}$?
3. How many yards are $\frac{27}{3}$ of a yard? $\frac{45}{5}$? $\frac{62}{3}$?
4. How many dollars are $\$\frac{21}{4}$? $\$\frac{36}{8}$? $\$\frac{75}{12}$? $\$\frac{84}{20}$?
5. In $\frac{90}{12}$ of a foot, how many feet? In $\frac{108}{9}$ of an acre, how many acres? In $\frac{180}{20}$ of a ton, how many tons?

WRITTEN EXERCISES.

213. 1. Reduce $\frac{218}{9}$ to a mixed number.

OPERATION.

$$\frac{218}{9} = 218 \div 9 = 24\frac{2}{9}$$

Analysis.—Since 9 ninths equal 1, 218 ninths are $24\frac{2}{9}$ times 9 ninths. Hence $\frac{218}{9} = 24\frac{2}{9}$.

2. Change $\frac{297}{12}$ to a mixed number.
3. In $\frac{407}{25}$ of a dollar, how many dollars?
4. How many rods in $\frac{525}{40}$ of a rod?

Rule.—*Divide the numerator by the denominator.*

Reduce to integers or mixed numbers,

5. $\frac{978}{56}$.
6. $\frac{2431}{85}$.
7. $\frac{738}{24}$.
8. $\frac{1512}{81}$.
9. $\frac{17321}{17}$.
10. $\frac{23280}{464}$.
11. $\frac{22625}{375}$.
12. $\frac{103407}{1046}$.
13. $\frac{2540304}{2468}$.

214. To reduce fractions to equivalent fractions having a common denominator.

ORAL EXERCISES.

1. How many *fourths* in 1 ? In $\frac{1}{2}$?
2. How many *ninths* in 1? In $\frac{1}{3}$? In $\frac{2}{3}$?
3. Express $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$, each as *twelfths.*
4. Change $\frac{3}{4}$ and $\frac{5}{8}$ to fractions of the same denominator.
5. What is a multiple of 4 ? Of 6? Of 8? Of 9?
6. What is a common multiple of 3 and 4 ? Of 4 and 5?
7. What is the least common multiple of 3, 4, and 6?
8. What is the least common multiple of the denominators of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$? Of $\frac{2}{3}$, $\frac{2}{9}$, and $\frac{5}{6}$?
9. Reduce $\frac{2}{3}$ and $\frac{1}{9}$ to eighteenths. To twenty-sevenths.
10. Name some fractions that can be changed to 16ths.
11. Name four fractions that can be changed to 24ths.

DEFINITIONS AND PRINCIPLES.

215. A ***Common Denominator*** is a denominator common to two or more fractions.

216. The ***Least Common Denominator*** of two or more fractions is the least denominator to which they can all be reduced.

Since all higher terms of a fraction are multiples of its corresponding lowest terms (**207,** Note), hence the following

217. PRINCIPLES.—1. *A common denominator of two or more fractions is a common multiple of their denominators.*

2. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

218. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ to equivalent fractions having a common denominator.

OPERATION.

$$2 \times 3 \times 5 = 30$$
$$\frac{1 \times 3 \times 5}{2 \times 3 \times 5} = \frac{15}{30}$$
$$\frac{2 \times 2 \times 5}{3 \times 2 \times 5} = \frac{20}{30}$$
$$\frac{3 \times 2 \times 3}{5 \times 2 \times 3} = \frac{18}{30}$$

ANALYSIS.—Multiply each denominator by the other two, and the product, 30, is a common denominator of the three. (PRIN. 1.)

But since the value of the fractions is not to be changed, each numerator must be multiplied by the same multiplier as its denominator. Hence, multiplying the terms of $\frac{1}{2}$ by 3 and 5, the result is $\frac{15}{30}$; of $\frac{2}{3}$, by 2 and 5, the result is $\frac{20}{30}$; and of $\frac{3}{5}$ by 2 and 3, the result is $\frac{18}{30}$. Or,

To find the numerators, take such part of the common denominator 30, as the given fraction is part of 1. Thus, $\frac{1}{2}$ of 30 is 15, etc.

Reduce to fractions having a common denominator

2. $\frac{3}{7}$ and $\frac{5}{9}$.
3. $\frac{7}{12}$ and $\frac{4}{5}$.
4. $\frac{3}{7}$, $\frac{5}{8}$, and $\frac{2}{3}$.
5. $\frac{7}{16}$, $\frac{2}{3}$, and $\frac{1}{4}$.
6. $\frac{9}{10}$, $\frac{5}{7}$, and $\frac{1}{2}$.
7. $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{2}{13}$.

8. Change $\frac{2}{3}$, $\frac{6}{7}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{r|rrr} 7 & 3 & 7 & 14 \\ \hline & 3 & 1 & 2 \end{array} \qquad \begin{array}{l} \frac{2}{3} = \frac{28}{42} \\ \frac{6}{7} = \frac{36}{42} \\ \frac{5}{14} = \frac{15}{42} \end{array}$$
$$2 \times 3 \times 7 = 42$$

ANALYSIS.—First find the least common multiple of the given denominators, which is 42. This must be the least common denominator of the given fractions. (PRIN. 2.)

9. Change $\frac{3}{5}$, $\frac{7}{12}$, and $\frac{11}{15}$ to equivalent fractions having the least common denominator.

RULE.—1. To reduce two or more fractions to equivalent fractions having a common denominator.

Multiply the terms of each fraction by the denominators of all the other fractions.

2. To reduce them to their *least* common denominator.

I. *Find the least common multiple of the denominators of the given fractions for their least common denominator.*

II. *Divide this common denominator by the denominator of each of the given fractions, and multiply its numerator by the quotient. The products are the new numerators.*

Mixed numbers must first be reduced to improper fractions.

Reduce to fractions having the least common denominator.

10. $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$.
11. $\frac{4}{13}$, $\frac{15}{26}$, and $\frac{7}{39}$.
12. $\frac{20}{21}$, $\frac{9}{56}$, and $\frac{51}{84}$.
13. $1\frac{11}{12}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$.
14. $\frac{3}{4}$, $2\frac{5}{7}$, $\frac{3}{5}$, and $1\frac{7}{10}$.
15. $6\frac{1}{4}$, $\frac{7}{20}$, 7, and $1\frac{1}{2}$.
16. $\frac{15}{40}$, $\frac{25}{120}$, $\frac{14}{64}$, and $2\frac{4}{5}$.
17. $\frac{57}{84}$, $\frac{67}{168}$, $\frac{59}{42}$, and $\frac{3}{21}$.

ADDITION.

ORAL EXERCISES.

219. 1. What is the sum of $\frac{3}{8}$ and $\frac{5}{8}$? Of $\frac{4}{9}$ and $\frac{7}{9}$?

2. How many times 1 is the sum of $\frac{6}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$?

3. Sold $\frac{4}{15}$ of an acre of land to one man, $\frac{7}{15}$ to another, and $\frac{8}{15}$ to a third. How much was sold to all?

How are fractions added that have a commen denominator

4. Mary paid \$$\frac{3}{4}$ for some ribbon, and \$$\frac{5}{6}$ for a pair of gloves. How much did she pay for both?

ANALYSIS.—She paid the sum of \$$\frac{3}{4}$ and \$$\frac{5}{6}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$; $\frac{9}{12}$ and $\frac{10}{12}$ are $\frac{19}{12}$, or $1\frac{7}{12}$. Hence she paid \$$1\frac{7}{12}$.

5. A man having $\frac{2}{5}$ of a ton of coal, bought $\frac{2}{3}$ of a ton more. How much had he then?

How are fractions added that have different denominators.

6. Henry gave \$$\frac{3}{8}$ for a book, \$$\frac{1}{4}$ for a slate, and \$$\frac{1}{2}$ for a bottle of ink. What did he pay for all?

7. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$? Of $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$?

8. Find the sum of $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12}$. Of $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{7}{10}$.

9. Find the sum of $\frac{7}{16}$, $\frac{3}{8}$, and $\frac{1}{4}$. Of $\frac{5}{9}$, $\frac{1}{4}$, and $\frac{7}{12}$.

10. A farmer sold $3\frac{1}{4}$ tons of hay to one man, and $5\frac{3}{8}$ to another. How much did he sell to both?

Analysis.—The sum of $3\frac{1}{4}$ tons and $5\frac{3}{8}$ tons. 5 and 3 are 8; and $\frac{1}{4}$ and $\frac{3}{8}$ are $\frac{5}{8}$, which added to 8 makes $8\frac{5}{8}$ tons. Hence, etc.

11. A man bought $5\frac{1}{2}$ cords of wood at one time, and $7\frac{9}{16}$ at another. How much did he buy in all?

How are mixed numbers added?

12. A man paid \$$25\frac{7}{8}$ for a watch, and sold it for \$$6\frac{1}{4}$ more than he gave for it. What did he sell it for?

Find the sum

13. Of $\frac{3}{4}$ and $3\frac{1}{2}$.
14. Of $5\frac{1}{3}$ and $\frac{7}{9}$.
15. Of $1\frac{5}{6}$ and $\frac{2}{3}$.
16. Of $2\frac{1}{2}$ and $6\frac{3}{8}$.
17. Of $8\frac{1}{7}$ and $\frac{9}{14}$.
18. Of $15\frac{1}{2}$ and $\frac{5}{3}$.
19. Of $2\frac{1}{4}$ and $1\frac{2}{3}$.
20. Of $5\frac{1}{3}$ and $\frac{5}{6}$.
21. Of $1\frac{9}{16}$ and $12\frac{5}{8}$.

220. Principles.—*Fractions can be added only when they have a common denominator, and when they express parts of like units.*

WRITTEN EXERCISES.

221. 1. Find the sum of $\frac{2}{5}$, $\frac{7}{12}$, and $\frac{4}{15}$.

OPERATION.

$$\frac{2}{5}+\frac{7}{12}+\frac{4}{15}=\frac{24+35+16}{60}$$

$$\frac{24+35+16}{60}=\frac{75}{60}=1\frac{1}{4}$$

Analysis.—Reduce the given fractions to equivalent fractions having the least common denominator, which is 60 (**217**, 2). Then add their *numerators,* and write the sum, 75, over the common denominator 60, *and* $\frac{75}{60}=1\frac{1}{4}$ *is the* required result.

2. What is the sum of $14\frac{3}{4}$, $25\frac{7}{8}$, and $7\frac{5}{12}$?

OPERATION.

$$\begin{aligned} 14\tfrac{3}{4} &= 14\tfrac{18}{24} \\ 25\tfrac{7}{8} &= 25\tfrac{21}{24} \\ 7\tfrac{5}{12} &= \underline{\ \ 7\tfrac{15}{24}} \\ & \quad 46 + \tfrac{54}{24} = 48\tfrac{1}{4} \end{aligned}$$

Analysis.—The sum of the fractions is $\frac{54}{24} = 2\frac{1}{4}$, which added to the sum of the integers 46, gives $48\frac{1}{4}$, the required sum.

Find the sum

3. Of $\frac{5}{12}$, $\frac{13}{15}$, and $\frac{7}{20}$.
4. Of $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{9}{2}$.
5. Of 42, $31\frac{5}{12}$, and $9\frac{7}{18}$.
6. Of $204\frac{7}{15}$, $50\frac{10}{21}$, and $7\frac{8}{35}$.

Rule.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers or integers, add the fractions and integers separately, then add the results.*

7. $\frac{1}{2}+\frac{2}{3}+\frac{4}{13}+\frac{9}{17}=$?
8. $\frac{3}{4}+7\frac{5}{6}+\frac{31}{42}+\frac{9}{14}=$?
9. $18\frac{5}{14}+2\frac{4}{7}+1\frac{11}{35}=$?
10. $\frac{5}{8}+6\frac{7}{12}+21\frac{5}{6}+77=$?
11. $\frac{15}{48}+\frac{11}{16}+7\frac{3}{8}+60+\frac{75}{96}=$?
12. $124\frac{4}{7}+325\frac{5}{12}+40\frac{10}{21}=$?

13. Bought 3 pieces of cloth containing $105\frac{7}{8}$, $86\frac{3}{4}$, and $58\frac{2}{3}$ yards respectively; how many yards in all?

14. If it takes $5\frac{1}{3}$ yards of cloth for a coat, $3\frac{1}{6}$ yards for a pair of pantaloons, and $\frac{7}{8}$ of a yard for a vest, how many yards does it take for all?

15. Four cheeses weighed respectively $46\frac{5}{8}$, $48\frac{2}{3}$, $49\frac{7}{16}$, and $57\frac{1}{4}$ pounds. What was their entire weight?

16. What number is that from which if $24\frac{4}{7}$ is taken, the remainder is $63\frac{22}{35}$?

17. A farm is divided into 4 fields: the first contains $29\frac{7}{12}$ acres, the second $50\frac{16}{21}$ acres, the third $41\frac{6}{7}$ acres, and the *fourth* $69\frac{3}{4}$ acres. How many acres in the farm?

SUBTRACTION.

ORAL EXERCISES.

222. 1. What is the difference between $\frac{6}{7}$ and $\frac{3}{7}$?

2. What is the difference between $\frac{3}{12}$ and $\frac{7}{12}$?

How is one fraction subtracted from another, each having the same denominator?

3. A gentleman who owned a sail-boat sold $\frac{5}{16}$ of it. What part did he still own?

4. A boy having $\$\frac{3}{4}$, gave $\$\frac{1}{3}$ for a neck-tie. What had he left?

ANALYSIS.—He had left the difference between $\$\frac{3}{4}$ and $\$\frac{1}{3}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{1}{3}$ equals $\frac{4}{12}$; $\frac{9}{12}$ less $\frac{4}{12}$ are $\frac{5}{12}$. Hence he had $\$\frac{5}{12}$ left.

5. A man owning $\frac{2}{3}$ of an acre of ground, sold $\frac{1}{6}$ of an acre. What part remained?

How is one fraction subtracted from another having a different denominator?

6. Subtract $\frac{1}{2}$ from $\frac{5}{8}$; $\frac{2}{5}$ from $\frac{1}{2}$; $\frac{3}{7}$ from $\frac{9}{14}$.

7. Find the difference between $\frac{7}{10}$ and $\frac{3}{5}$; $\frac{2}{3}$ and $\frac{7}{8}$.

8. From a piece of cloth containing $12\frac{1}{2}$ yards, $5\frac{1}{8}$ yards were cut. How many yards remained?

ANALYSIS.—The difference between $12\frac{1}{2}$ yards and $5\frac{1}{8}$ yards. $\frac{1}{8}$ from $\frac{1}{2}$ leaves $\frac{3}{8}$, and 5 from 12 leaves 7. Hence $7\frac{3}{8}$ yards remained.

9. If a ton of coal costs $\$7\frac{4}{5}$, and a cord of wood $\$4\frac{1}{3}$, what is the difference in their cost?

How is one mixed number subtracted from another?

10. What is the value of $3\frac{1}{2}-2\frac{1}{5}$? $8\frac{1}{4}-2\frac{1}{6}$? $6\frac{11}{12}-\frac{2}{3}$?

223. PRINCIPLE.—*Fractions can be subtracted only when they have a common denominator, and when they express parts of like units.*

WRITTEN EXERCISES.

224. 1. From $\frac{7}{9}$ subtract $\frac{4}{15}$.

OPERATION.

$$\tfrac{7}{9} - \tfrac{4}{15} = \tfrac{35}{45} - \tfrac{12}{45} = \tfrac{35-12}{45} = \tfrac{23}{45}$$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator. Hence $\frac{35}{45} - \frac{12}{45} = \frac{23}{45}$.

2. From $134\frac{1}{3}$ take $76\frac{5}{8}$.

OPERATION.

$$\begin{array}{rl} 134\tfrac{1}{3} = & 134\tfrac{8}{24} \\ 76\tfrac{5}{8} = & \underline{\ \ 76\tfrac{15}{24}} \\ & \ \ 57\tfrac{17}{24} \end{array}$$

ANALYSIS.—Reduce $\frac{1}{3}$ and $\frac{5}{8}$ to equivalent fractions having the least common denominator. As $\frac{15}{24}$ cannot be taken from $\frac{8}{24}$, take 1 or $\frac{24}{24}$ from 134, leaving 133, and add it to $\frac{8}{24}$, making $\frac{32}{24}$. Then $\frac{15}{24}$ from $\frac{32}{24}$ leaves $\frac{17}{24}$, and 76 from 133, leaves 57. Hence $57\frac{17}{24}$ is the result.

3. From $\frac{9}{14}$ take $\frac{16}{63}$.
4. From $\frac{7}{12}$ take $\frac{5}{42}$.
5. From $\frac{24}{35}$ take $\frac{7}{24}$.
6. From $36\frac{5}{7}$ take $10\frac{1}{2}$.
7. From $112\frac{7}{102}$ take 56.
8. From $204\frac{3}{16}$ take $39\frac{7}{12}$.

RULE.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, and write the difference of the numerators over the common denominator.*

II. *When there are mixed numbers, subtract the fractional and integral parts separately, and add the results.*

If the mixed numbers are small, they may be reduced to improper fractions and subtracted according to the usual method.

Find the difference between

9. $\frac{5}{8}$ and $\frac{3}{11}$.
10. $1\frac{3}{4}$ and $\frac{5}{6}$.
11. $\frac{14}{15}$ and $2\frac{4}{5}$.
12. 16 and $3\frac{2}{11}$.
13. $63\frac{7}{9}$ and $71\frac{1}{8}$.
14. 106 and $95\frac{15}{36}$.

15. From $\frac{7}{64}$ take $\frac{13}{120}$.
16. From $16\frac{7}{15}$ take $\frac{43}{125}$.
17. From $410\frac{5}{27}$ take $226\frac{5}{9}$.
18. From $428\frac{11}{12}$ take $180\frac{10}{11}$.

19. A farmer having 208 acres of land, sold $92\frac{7}{25}$ acres. How many acres had he left?

ADDITION AND SUBTRACTION.

ORAL EXERCISES.

225. 1. How much less than 2, is $\frac{1}{4}+\frac{2}{3}$?

2. How much greater than 2, is $\frac{2}{3}+\frac{1}{2}+1\frac{1}{6}$?

3. What is the difference between 4 and $2\frac{3}{8}$? $5\frac{1}{7}$ and $7\frac{5}{7}$?

4. Mr. Smith sold $\frac{1}{3}$ of his farm to one man, $\frac{1}{4}$ to another, and $\frac{1}{8}$ to a third. What part had he left?

5. Paid \$$6\frac{1}{2}$ for a ton of coal, and \$$3\frac{1}{4}$ for a load of wood. What change must be returned for a ten-dollar bill?

6. What is the difference between $1\frac{4}{5}+\frac{2}{3}$ and $5\frac{8}{15}$?

7. What is the difference between $\frac{9}{10}+\frac{1}{2}$ and $\frac{1}{3}+\frac{5}{6}$?

Find the second member of the following equations:

8. $\frac{5}{6}+\frac{1}{3}-\frac{1}{2}=?$
9. $\frac{9}{10}-\frac{1}{5}+2=?$
10. $3-(\frac{4}{5}-\frac{1}{2})=?$
11. $6\frac{1}{2}+\frac{4}{5}-1\frac{3}{4}=?$
12, $12-(8-2\frac{5}{8})+1\frac{1}{2}=?$
13. $(26-14)-(\frac{3}{4}+3\frac{3}{8})=?$
14. $(1\frac{3}{8}+2\frac{3}{4})-(\frac{1}{2}+\frac{2}{3})=?$
15. $(5-3\frac{5}{7})+(9\frac{3}{14}-6)=?$
16. $8\frac{9}{16}-\overline{2\frac{1}{4}+\frac{7}{8}}+10=?$
17. $12\frac{7}{20}+\overline{9\frac{1}{2}-1\frac{4}{5}}-2\frac{1}{4}=?$

WRITTEN EXAMPLES.

226. 1. The sum of two numbers is $124\frac{1}{4}$, and the less is $36\frac{2}{13}$. What is the greater?

2. What number added to $147\frac{4}{7}$ will make $216\frac{3}{8}$?

3. What number added to $307\frac{1}{7}+210\frac{3}{4}$ will make $700\frac{5}{8}$?

4. What number must be added to the difference of $186\frac{5}{9}$ and $214\frac{3}{7}$ to make $1042\frac{12}{21}$?

5. What fraction added to the sum of $\frac{1}{8}$, $\frac{5}{12}$, and $\frac{5}{18}$, will make $\frac{133}{144}$?

6. What must be added to $\frac{4}{5}$, that the sum may be $\frac{17}{21}$?

7. Bought a quantity of barrel staves for \$$160\frac{3}{8}$, and of lumber for \$$1136\frac{2}{3}$. Sold the staves for \$$205\frac{1}{2}$ and the lumber for \$$1240\frac{9}{16}$. What was the whole gain?

8. A man bought a ton of hay for \$$15\frac{3}{8}$, a barrel of flour for \$$9\frac{5}{12}$, and a barrel of apples for \$$3\frac{7}{16}$. What change should be returned to him for 3 ten-dollar bills?

Complete the following equations:

9. $\overline{\frac{4}{3}+\frac{6}{7}}-\overline{\frac{3}{8}+\frac{5}{7}}=?$
10. $8\frac{5}{6}+2\frac{2}{3}-5\frac{4}{11}=?$
11. $48-(16\frac{4}{7}-3\frac{1}{2})=?$
12. $41\frac{1}{3}+\overline{56-24\frac{7}{30}}-4\frac{11}{15}=?$
13. $\overline{120-51\frac{2}{9}}+\overline{90\frac{10}{27}}-\frac{2}{3}=?$
14. $342-(\overline{21\frac{3}{13}+\frac{4}{5}}-9)=?$
15. $176\frac{9}{20}+(132\frac{7}{8}-26\frac{21}{40})-17\frac{1}{2}=?$
16. \$$1000-($\$$500-$\$$107\frac{55}{100})+$\$$91\frac{9}{10}=?$

MULTIPLICATION.

227. When one factor is a fractional number.

ORAL EXERCISES.

1. What part of a mile is 3 times $\frac{1}{4}$ of a mile?
2. What part of a dollar is 4 times $\frac{1}{8}$ of a dollar?
3. How many times 1 is 3 times $\frac{2}{3}$? 4 times $\frac{3}{4}$?
4. At \$$\frac{5}{8}$ a pound, what will 4 pounds of tea cost?

ANALYSIS.—Four pounds will cost 4 times \$$\frac{5}{8}$, or \$$\frac{20}{8}$, equal to \$$2\frac{1}{2}$. Hence 4 pounds of tea will cost \$$2\frac{1}{2}$.

5. At \$$\frac{5}{6}$ a bushel, what is the cost of 6 bushels of oats? Of 7 bushels? Of 8 bushels? Of 9 bushels?

6. If a horse eat $\frac{4}{7}$ of a bushel of grain in a day, how much will 4 horses eat? 6 horses? 8 horses? 10 horses?

7. What cost 12 baskets of pears, at \$$\frac{4}{5}$ a basket?

With each class of *oral* questions in Art. **227**, the pupil may solve the corresponding *written* examples on pages 118 and 119.

8. What cost 9 pounds of butter at $$\frac{3}{8}$ a pound?

9. Show that multiplying the numerator of $\frac{3}{20}$ by 4 multiplies the fraction by 4.

10. Show that dividing the denominator of $\frac{3}{20}$ by 4 multiplies the fraction by 4.

How many ways to multiply a fraction by an integer?

11. Multiply $\frac{3}{11}$ by 5; $\frac{4}{18}$ by 6; $\frac{9}{20}$ by 5; $\frac{7}{40}$ by 8.

12. At 4\frac{3}{4}$ a box, what will 5 boxes of raisins cost?

Analysis.—They will cost 5 times 4\frac{3}{4}$. 5 times $$\frac{3}{4}$ are 3\frac{3}{4}$, and 5 times $4 are $20. $20 + 3\frac{3}{4}$ = 23\frac{3}{4}$. Hence, etc.

13. At 7$\frac{1}{2}$ cents a pound, what will 9 pounds of rice cost?

14. At 9\frac{4}{5}$ a barrel, what is the cost of 6 barrels of flour? Of 8 barrels? Of 9 barrels? Of 10 barrels?

15. What will 8 yards of cloth cost, at 5\frac{7}{10}$ a yard?

16. Multiply 7$\frac{1}{6}$ by 9; 9$\frac{4}{7}$ by 6; 10$\frac{5}{8}$ by 7; 12$\frac{7}{10}$ by 8.

17. What is $\frac{1}{2}$ of 12 yards? $\frac{1}{3}$ of 24 men? $\frac{1}{4}$ of $30?

18. Multiplying by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, etc., is the same as dividing by what integers?

When a *fractional part* of an integer, or of a fraction, is to be taken, the word *of*, and not *times*, should be used.

19. At $7 a ton, what will $\frac{3}{4}$ of a ton of coal cost?

Analysis.—It will cost $\frac{3}{4}$ of $7, or 3 times $\frac{1}{4}$ of $7. $\frac{1}{4}$ of $7 is 1\frac{3}{4}$, and 3 times 1\frac{3}{4}$ are 5\frac{1}{4}$. Hence, etc.

20. At $12 a gross, what will $\frac{7}{8}$ of a gross of butts cost?

21. What will $\frac{9}{10}$ of a ton of hay cost, at $15 a ton?

22. What is $\frac{5}{8}$ of 6? $\frac{4}{5}$ of 2? $\frac{2}{9}$ of 8? $\frac{5}{12}$ of 9?

23. At $5 a yard, what will $\frac{4}{5}$ of a yard of cloth cost?

24. If a man can build a wall in 28 days, in what time can he build $\frac{2}{3}$ of it? $\frac{3}{4}$ of it? $\frac{5}{8}$ of it?

25. If an acre of land produce 45 bushels of corn, how *much will $\frac{2}{3}$ of an acre* produce? $\frac{5}{8}$? $\frac{3}{4}$? $\frac{4}{9}$? $\frac{9}{10}$?

26. What is $\frac{2}{3}$ of \$5? Of \$7? Of \$16? Of \$25?

27. Multiply 50 by $\frac{3}{4}$; 49 by $\frac{6}{7}$; 63 by $\frac{5}{21}$; 81 by $\frac{7}{10}$.

28. In \$1 are 100 cents; how many cents in $\frac{1}{2}$ of a dollar? In $\frac{1}{4}$? $\frac{1}{8}$? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{20}$?

29. How many cents in $\frac{3}{10}$ of a dollar? In $\frac{3}{20}$? $\frac{3}{4}$? $\frac{5}{8}$?

30. Which is greater, $\frac{4}{5}$ of 15, or 15 × $\frac{4}{5}$?

31. Show that a fraction of an integer equals the product of the integer by the fraction.

How is an integer multiplied by a fraction?

32. At \$12 a ton, what will $5\frac{3}{8}$ tons of cheese cost?

Analysis.—It will cost $5\frac{3}{8}$ times \$12. 5 times \$12 are \$60, and $\frac{3}{8}$ of \$12 are $\$4\frac{1}{2}$, which added to \$60, make $\$64\frac{1}{2}$. Hence, etc.

33. At 15 cents each, what will $4\frac{3}{4}$ melons cost?

34. What will $7\frac{2}{3}$ weeks' board cost, at \$9 a week?

35. How much is $6\frac{2}{3}$ times 12? $5\frac{4}{5}$ times 20?

36. Multiply 4 by $8\frac{1}{6}$; 6 by $7\frac{5}{8}$; 8 by $9\frac{3}{10}$.

What

37. Is $\frac{7}{12}$ of 10 gallons?
38. Is $\frac{3}{8}$ of 47 pounds?
39. Is $\frac{5}{7}$ of 90 rods?
40. Is $\frac{7}{9}$ of 56 days?

How many

41. Are 10 times $6\frac{4}{5}$ tons?
42. Are 9 times $11\frac{2}{13}$ miles?
43. Are $7\frac{3}{4}$ times 12 men?
44. Are $12\frac{1}{2}$ times 9 minutes?

Find the value

45. Of $\frac{3}{9} \times 15$.
46. Of $28 \times \frac{6}{7}$.
47. Of $56 \times \frac{7}{9}$.
48. Of $\frac{4}{21} \times 7$.
49. Of $\frac{5}{6} \times 7 + \frac{1}{8}$.
50. Of $\frac{7}{5} \times 9 - 2\frac{1}{5}$.
51, Of $27 \times \frac{5}{8} + 3\frac{1}{4}$.
52. Of $6\frac{2}{7} \times 7 - 1\frac{7}{8}$.
53. Of $\frac{7}{10} + \overline{\frac{2}{3} \times 5}$.
54. Of $1\frac{4}{15} - \overline{2 \times \frac{1}{5}}$.
55. Of $4\frac{5}{18} + \overline{2 \times 5\frac{5}{9}}$.
56. Of $9 \times 9\frac{1}{7} + 20\frac{1}{2}$.

228. Principles.—1. *Multiplying the numerator or dividing the denominator multiplies the fraction.* (**200,** 1.)

2. *The product of an integer by a fraction is equal to such part of the integer as the fraction is of a unit.*

WRITTEN EXERCISES.

229. 1. Multiply $\frac{7}{27}$ by 9.

OPERATION.

$\frac{7}{27}\times 9=\frac{7\times 9}{27}=\frac{63}{27}=2\frac{1}{3}$

Or,

$\frac{7}{27}\times 9=\frac{7}{27\div 9}=\frac{7}{3}=2\frac{1}{3}$

Or,

$$\begin{array}{r|l} {}^{3}\,\cancel{27} & 7 \\ & \cancel{9} \\ \hline 3 & 7 \\ \cline{2-2} & 2\frac{1}{3} \end{array}$$

ANALYSIS.—Multiply the numerator 7 by 9, or divide the denominator 27 by 9; either operation will give $2\frac{1}{3}$, the required result. (PRIN. 1.)

By using the vertical line and cancellation, both operations are combined and shortened.

In the first operation, the *number* of parts or of fractional units is increased, while their *size* or value remains the same: in the second operation, the *size* of the parts is increased, while their *number* remains unchanged.

In like manner multiply

2. $\frac{9}{14}$ by 12.
3. $\frac{17}{63}$ by 9.
4. $\frac{15}{17}$ by 13.
5. $\frac{7}{75}$ by 15.
6. $\frac{45}{120}$ by 36.
7. $\frac{51}{168}$ by 21.
8. $\frac{4}{102}$ by 17.
9. $\frac{101}{132}$ by 22.
10. $\frac{54}{99}$ by 44.

11. Multiply 72 by $\frac{4}{9}$.

OPERATION.

$72\times\frac{4}{9}=\overline{72\div 9}\times 4=32$

Or, $72\times\frac{4}{9}=\frac{\overset{8}{\cancel{72}}\times 4}{\cancel{9}}=32$

Or,

$$\begin{array}{r|l} & \cancel{72}\,{}^{8} \\ \cancel{9} & 4 \\ \hline & 32 \end{array}$$

ANALYSIS.—To multiply 72 by $\frac{4}{9}$, is to find $\frac{4}{9}$ of 72. $\frac{4}{9}$ of 72 is 4 times $\frac{1}{9}$ of 72, which is 32. (PRIN. 2.)

What is the product

12. Of 75 by $\frac{3}{15}$.
13. Of 7 by $\frac{8}{21}$.
14. Of 56 by $\frac{5}{14}$.
15. Of 168 by $\frac{33}{44}$.
16. Of 200 by $\frac{9}{14}$.
17. Of 315 by $\frac{17}{21}$.
18. Of 19 by $\frac{13}{47}$.
19. Of 448 by $\frac{9}{56}$.
20. Of 572 by $\frac{5}{24}$.

A fraction is *multiplied* by a number equal to its *denominator* by *cancelling* the denominator. Thus, $\frac{7}{8}\times 8=7$.

Cancelling a *factor* of the denominator *multiplies* the fraction by *that factor. Thus,* $\frac{5}{12}\times 4=\frac{5}{3}$.

21. Multiply $17\frac{2}{9}$ by 6.

OPERATION.

$$\begin{array}{r} 17\frac{2}{9} \\ 6 \\ \hline 103\frac{1}{3} \end{array} \qquad \text{Or, } 17\frac{2}{9} = \frac{155}{9} \qquad \begin{array}{r|l} {}^{3}\cancel{9} & 155 \\ & \cancel{6}^{2} \\ \hline 3 & 310 \\ \hline & 103\frac{1}{3} \end{array}$$

Analysis.—To multiply $17\frac{2}{9}$ by 6, multiply the fraction $\frac{2}{9}$, and the integer 17 separately and add their products, which gives $103\frac{1}{3}$, the required product. Or,

Reduce the mixed number to an improper fraction, and multiply as in Ex. 1, which gives the same result.

Multiply

22. $127\frac{2}{3}$ by 12.
23. $85\frac{4}{5}$ by 15.
24. $128\frac{5}{21}$ by 42.
25. $246\frac{5}{6}$ by 16.
26. $314\frac{9}{16}$ by 48.
27. $750\frac{9}{25}$ by 17.

28. Multiply 140 by $9\frac{2}{3}$.

OPERATION.

$$\begin{array}{r} 140 \\ 9\frac{2}{3} \\ \hline 93\frac{1}{3} \\ 1260 \\ \hline 1353\frac{1}{3} \end{array} \qquad \text{Or, } 9\frac{2}{3} = \frac{29}{3} \qquad \begin{array}{r|l} & 140 \\ 3 & 29 \\ \hline 3 & 4060 \\ \hline & 1353\frac{1}{3} \end{array}$$

Analysis.—To multiply 140 by $9\frac{2}{3}$, multiply by the fraction $\frac{2}{3}$, and by the integer 9, separately, and add their products, which gives $1353\frac{1}{3}$, the required product. Or,

Reduce the mixed number to an improper fraction, and multiply as in Ex. 1, which gives the same result.

Multiply

29. 96 by $12\frac{5}{8}$.
30. 216 by $16\frac{7}{9}$.
31. 304 by $24\frac{7}{16}$.
32. 198 by $18\frac{5}{6}$.
33. 560 by $23\frac{7}{10}$.
34. 715 by $14\frac{4}{15}$.

35. Multiply $327\frac{7}{12}$ by 72; 2466 by $84\frac{3}{4}$; 759 by $\frac{14}{33}$.

36. What will 120 dozen of hose cost at \$$4\frac{5}{6}$ a dozen?

37. At \$20 a ton, what will $\frac{19}{28}$ of a ton of hay cost?

38. If a city lot is worth \$3145, what is $\frac{9}{16}$ of it worth?

39. What will 142 yards of curbing cost at \$$6\frac{5}{8}$ a yard?

40. At \$$1\frac{5}{16}$ a yard, what is the cost of 8 yards of cloth? Of 24 yards? Of 64 yards? Of 120 yards?

230. When both factors are fractional numbers.

ORAL EXERCISES.

1. A boy having $\frac{1}{2}$ of a melon, gave $\frac{1}{2}$ of it to his sister. What part of the melon did she receive?

2. What part of 1 is $\frac{1}{2}$ of $\frac{1}{2}$? Is $\frac{1}{3}$ of $\frac{1}{2}$? Is $\frac{1}{4}$ of $\frac{1}{2}$?

3. What part of 1 is $\frac{1}{2}$ of $\frac{4}{5}$? $\frac{1}{3}$ of $\frac{6}{7}$? $\frac{1}{4}$ of $\frac{8}{9}$? $\frac{1}{5}$ of $\frac{5}{7}$?

4. Which is greater, $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{4}{5}$, or $\frac{1}{3}$ of $\frac{3}{5}$? $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{6}$ of $\frac{1}{2}$?

5. If I own $\frac{6}{7}$ of an acre of land, and sell $\frac{1}{3}$ of it, what part of an acre do I sell? What part do I retain?

6. If a yard of silk is worth $\$\frac{3}{8}$, what is $\frac{1}{3}$ of a yard worth?

7. A boy having $\$\frac{3}{4}$ gave $\frac{5}{6}$ of it for a knife. What part of a dollar did he pay for the knife?

ANALYSIS.—He paid $\frac{5}{6}$ of $\$\frac{3}{4}$, or 5 times $\frac{1}{6}$ of $\$\frac{3}{4}$. $\frac{1}{6}$ of $\$\frac{3}{4}$ is $\$\frac{3}{24}$, and 5 times $\$\frac{3}{24}$ are $\frac{15}{24}$, or $\$\frac{5}{8}$. Hence, etc.

8. At $\$\frac{4}{5}$ a gallon, what will $\frac{3}{4}$ of a gallon of syrup cost?

Fractions with the word *of* between them are sometimes called *Compound Fractions*. The word *of* is equivalent to the sign (×) of multiplication. Thus, $\frac{5}{9}$ *of* $\frac{2}{3} = \frac{5}{9} \times \frac{2}{3}$; $\frac{4}{5}$ *of* $9 = \frac{4}{5} \times 9$, etc.

9. What is $\frac{2}{3}$ of $\frac{6}{7}$? $\frac{4}{5}$ of $\frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{8}$? $\frac{3}{7}$ of $\frac{5}{2}$?

10. What is $\frac{4}{7} \times \frac{3}{5}$? $\frac{9}{10} \times \frac{2}{5}$? $\frac{3}{8} \times \frac{5}{12}$? $\frac{7}{9} \times \frac{3}{14}$?

11. A man owning $\frac{4}{5}$ of a mill, sold $\frac{5}{8}$ of his share to his brother. What part of the mill did each then own?

12. At $\$8\frac{4}{5}$ a barrel, what will $\frac{3}{4}$ of a barrel of flour cost?

ANALYSIS.—It will cost $\frac{3}{4}$ of $\$8\frac{4}{5}$, or 3 times $\frac{1}{4}$ of $\$8\frac{4}{5}$. $\frac{1}{4}$ of $\$8\frac{4}{5}$ is $\$2\frac{1}{5}$, and 3 times $\$2\frac{1}{5}$ are $\$6\frac{3}{5}$. Hence, etc.
Or, $\$8\frac{4}{5}$ equal $\$\frac{44}{5}$, and $\frac{3}{4}$ of $\$\frac{44}{5} = \$\frac{33}{5} = \$6\frac{3}{5}$.

13. At $\$9\frac{1}{4}$ a case, what will $\frac{2}{3}$ of a case of slates cost?

14. At $\$5\frac{1}{2}$ a yard, what will $\frac{4}{5}$ of a yard of cloth cost?

15. How much is $\frac{5}{6}$ of $7\frac{1}{2}$ miles? $\frac{3}{8}$ of $24\frac{1}{4}$ pounds?

16. If a man travel $26\frac{1}{3}$ miles in a day, how far does he travel in $\frac{1}{4}$ of a day? In $\frac{3}{4}$? In $\frac{5}{6}$? In $\frac{3}{8}$?

17. At $\$\frac{2}{3}$ a yard, what will $6\frac{1}{4}$ yards of flannel cost?

ANALYSIS.—It will cost $6\frac{1}{4}$ times $\$\frac{2}{3}$. 6 times $\$\frac{2}{3}$ are \$4, and $\frac{1}{4}$ of $\$\frac{2}{3}$ is $\$\frac{1}{6}$, which added to \$4, makes $\$4\frac{1}{6}$. Hence, etc.

Or, $6\frac{1}{4}=\frac{25}{4}$, and $\frac{25}{4}$ of $\$\frac{2}{3}=\$\frac{25}{6}=\$4\frac{1}{6}$.

18. At $\$\frac{7}{10}$ a pound, what will $8\frac{2}{3}$ pounds of tea cost?

19. What will $9\frac{3}{8}$ pounds of beef cost, at $\$\frac{1}{5}$ a pound?

20. If a man hoe $\frac{7}{8}$ of an acre of corn in a day, how many acres can he hoe in $5\frac{1}{2}$ days? In $6\frac{3}{4}$ days? In $8\frac{4}{5}$ days?

21. What will $2\frac{1}{4}$ yards of silk cost, at $\$3\frac{1}{3}$ a yard?

ANALYSIS.—$2\frac{1}{4}=\frac{9}{4}$, and $3\frac{1}{3}=\frac{10}{3}$; and $\frac{10}{3}\times\frac{9}{4}=\frac{15}{2}=\$7\frac{1}{2}$, Hence, etc.

Reduce mixed numbers to improper fractions, and then proceed as in multiplying one fraction by another.

22. What cost $5\frac{3}{8}$ yards of alpaca, at $\$1\frac{2}{3}$ a yard?

23. Multiply $2\frac{1}{2}$ by $3\frac{1}{5}$; $7\frac{1}{3}$ by $1\frac{3}{4}$; $5\frac{1}{6}$ by $2\frac{2}{3}$.

Find the value

24. Of $\frac{3}{4}$ of $\frac{8}{9}$ of a mile.
25. Of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of \$1.
26. Of $\frac{3}{5}$ of $4\frac{7}{10}$ leagues.
27. Of $\frac{5}{4}$ of $6\frac{2}{5}$ pecks.
28. Of $7\frac{1}{4}$ times $\$\frac{5}{8}$.
29. Of $9\frac{2}{5}$ times $\frac{3}{7}$ of a rod.
30. Of $2\frac{1}{4}$ times $5\frac{1}{2}$ acres.
31. Of $4\frac{1}{2}$ times $\frac{3}{4}$ of $2\frac{1}{2}$ feet.

Find the result of

32. $\frac{3}{7}\times\frac{3}{4}$.
33. $1\frac{3}{4}\times\frac{5}{8}$.
34. $\frac{7}{10}\times6\frac{1}{7}$.
35. $4\frac{1}{9}\times1\frac{2}{3}$.
36. $10-\frac{4}{5}$ of $3\frac{1}{4}$.
37. $6\frac{7}{8}+\overline{2\frac{1}{2}\times\frac{5}{6}}$.
38. $\frac{15}{16}-\frac{1}{8}$ of $2\frac{3}{4}$.
39. $\frac{11}{12}\times\frac{4}{3}+8\frac{7}{9}$.
40. $2\frac{1}{4}\times\frac{5}{6}-\frac{3}{8}$ of $1\frac{2}{3}$.
41. $16-\overline{\frac{3}{10}\times12\frac{2}{3}}$.
42. $4\frac{1}{8}+\frac{5}{4}$ of $1\frac{1}{2}-\frac{7}{16}$.
43. $\frac{18}{33}$ of $\frac{11}{9}+\overline{2\frac{1}{3}\times0}$.

231. PRIN.—*The product of a fraction by a fraction is such a part of either factor as the other is of a unit.*

WRITTEN EXERCISES.

232. 1. Multiply $\frac{5}{14}$ by $\frac{7}{8}$.

OPERATION.

$$\frac{5}{14}\times\frac{7}{8}=\frac{35}{112}=\frac{5}{16}$$

$$\text{Or, }\frac{5}{\underset{2}{\cancel{14}}}\times\frac{\cancel{7}}{8}=\frac{5}{16}$$

ANALYSIS.—To multiply $\frac{5}{14}$ by $\frac{7}{8}$ is to find $\frac{7}{8}$ of $\frac{5}{14}$, which is 7 times $\frac{1}{8}$ of $\frac{5}{14}$, and $\frac{1}{8}$ is found by multiplying the denominator (**200**, 1.) Hence $\frac{1}{8}$ of $\frac{5}{14}$ is $\frac{5}{112}$, and $\frac{7}{8}$ of $\frac{5}{14}$ is 7 times $\frac{5}{112}$, or $\frac{35}{112}=\frac{5}{16}$, the required product.

In like manner multiply

2. $\frac{11}{24}$ by $\frac{36}{57}$.
3. $\frac{20}{33}$ by $\frac{11}{15}$.
4. $\frac{7}{12}$ by $\frac{17}{42}$.
5. $\frac{15}{17}$ by $\frac{51}{75}$.
6. $\frac{45}{126}$ by $\frac{7}{860}$.
7. $\frac{9}{56}$ by $\frac{128}{153}$.

8. Find the product of $\frac{7}{16}$, $\frac{4}{21}$, $5\frac{1}{3}$, and 2.

OPERATION.

$$\frac{\cancel{7}}{\cancel{16}}\times\frac{4}{\underset{3}{\cancel{21}}}\times\frac{\cancel{16}}{3}\times\frac{2}{1}=\frac{8}{9}$$

Or,

$$\begin{array}{r|l} \cancel{16} & \cancel{7} \\ {}^{3}\cancel{21} & 4 \\ 3 & \cancel{16} \\ & 2 \\ \hline 9 & 8=\frac{8}{9} \end{array}$$

ANALYSIS.—Change the mixed number $5\frac{1}{3}$ to an improper fraction, and the integer 2 to the form of a fraction, then multiply as in Ex. 1.

Find the value of the following expressions :

9. $\frac{8}{15}$ of $12\frac{1}{4}\times\frac{1}{5}$ of $7\frac{1}{3}$.
10. $\frac{2}{7}$ of $96\times\frac{7}{10}$ of $26\frac{2}{3}$.
11. $\frac{3}{4}$ of $\frac{4}{5}$ of $21\frac{1}{3}\times 2\frac{1}{2}$.
12. $42\frac{1}{2}\times 5\frac{1}{3}$ times $6\frac{3}{4}$.
13. $\frac{4}{9}$ of $36\frac{1}{2}\times\frac{3}{7}$ of 9.
14. $\frac{4}{5}$ of $91\times 72\frac{1}{2}$.
15. $\frac{7}{11}$ of $63\frac{1}{2}$ by $\frac{4}{5}$ of $\frac{55}{98}$.
16. $\frac{6}{63}$ of $21\times\frac{6}{19}$ of $\frac{11}{8}$ of $\frac{3}{5}$.

From the preceding principles and operations is derived the following *general*

RULE.—I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Multiply together the numerators, for the numerator; and the denominators, for the denominator of the product. Or,*

I. *Multiply by the numerator of the fractional multiplier and divide by the denominator.*

II. *When the multiplier is a mixed number, multiply by the fractional and integral parts separately and add their products.*

Cancel all factors common to numerators and denominators before multiplying, thus *shortening* the operation, and obtaining the answer in its *lowest terms*.

Find the cost,

17. Of 15 cords of bark, at $\$4\frac{5}{8}$ a cord.
18. Of $24\frac{3}{4}$ pounds of tea, at $\$\frac{7}{8}$ a pound.
19. Of 80 yards of cloth, at $\$4\frac{5}{16}$ a yard.
20. Of $21\frac{3}{5}$ bushels of corn, at $\$\frac{9}{16}$ a bushel.
21. Of $\frac{3}{4}$ of $5\frac{1}{2}$ tons of hay, at $\$15\frac{3}{10}$ a ton.
22. Of $18\frac{2}{3}$ barrels of crude oil, at $\$7\frac{4}{5}$ a barrel.
23. Of $\frac{1}{3}$ of $18\frac{1}{2}$ yards of silk, at $\frac{3}{4}$ of $5 a yard.
24. Of 126 pounds of beef, at $9\frac{1}{4}$ cents a pound.
25. Of $36\frac{7}{10}$ tons of railroad iron, at $\$62\frac{1}{2}$ a ton.
26. Of 35 horses, at $\$205\frac{3}{5}$ each.
27. Of $\frac{4}{5}$ of $156\frac{2}{3}$ acres of land, at $\frac{5}{6}$ of $\$54\frac{9}{10}$ an acre.
28. Of $28\frac{5}{8}$ bushels of sweet potatoes, at $\$1\frac{2}{3}$ a bushel.
29. Of $\frac{5}{8}$ of a yard of satin, at $\$\frac{13}{20}$ a yard.
30. Of $\frac{9}{40}$ of an acre of land, at $125 an acre.
31. Of $7\frac{9}{20}$ tons of middlings, at $\$26\frac{4}{5}$ a ton?
32. Paid $\$365\frac{7}{8}$ for a horse, and sold him for $\frac{4}{5}$ of what he cost. What was the loss?
33. When peaches are worth $\$\frac{7}{8}$ a basket, what are $126\frac{3}{4}$ baskets worth?
34. Find the value of $(129 - 76\frac{4}{5}) \times \frac{7}{12}$ of $\overline{12\frac{1}{4} - 2\frac{2}{3}} + (21\frac{1}{2} \times 6\frac{1}{3})$.

DIVISION.

233. When the divisor is an integral number.

ORAL EXERCISES.

1. If $\frac{3}{4}$ of an acre of land is divided into 3 equal lots, what part of an acre does each lot contain?

2. $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1? $\frac{1}{2}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{7}{8}$? $\frac{1}{5}$ of $\frac{5}{12}$?

3. Dividing by 3, 4, and 5 is the same as multiplying by what fractions?

4. How much is $\frac{9}{12} \times \frac{1}{3}$? $\frac{9}{12} \div 3$? $\frac{8}{15} \times \frac{1}{4}$? $\frac{8}{15} \div 4$?

5. If 4 slates cost \$$\frac{8}{9}$, what will 1 slate cost?

6. If 5 boxes of figs cost \$$\frac{3}{4}$, what will 1 box cost?

7. Divide $\frac{7}{8}$ of a barrel of flour equally among 3 families. What part of a barrel will each family receive?

8. What is $\frac{1}{8}$ of $\frac{9}{10}$? $\frac{9}{10}$ divided by 8?

9. What is the quotient of $\frac{18}{25}$ divided by 2? by 3?

10. Show that dividing the numerator of $\frac{9}{10}$ by 3 divides the fraction by 3.

11. Show that multiplying the denominator of $\frac{6}{7}$ by 3 divides the fraction by 3.

How many ways to divide a fraction by an integer?

12. Divide $\frac{8}{15}$ by 4; $\frac{10}{21}$ by 5; $\frac{3}{11}$ by 7; $\frac{5}{6}$ by 9.

13. If 6 pounds of sugar cost \$$\frac{3}{5}$, what will 1 pound cost?

14. Divide $\frac{5}{8}$ by $\frac{3}{5}$ of 20; $\frac{12}{17}$ by $\frac{2}{5}$ of 15; $\frac{7}{9}$ by $\frac{1}{8}$ of 40.

15. At \$4 a yard, how many yards of silk can be bought for \$$21\frac{5}{7}$?

ANALYSIS.—As many yards as \$4 is contained times in \$$21\frac{5}{7}$, or $\frac{1}{4}$ of $21\frac{5}{7} = \frac{1}{4}$ of $\frac{152}{7}$, or $\frac{38}{7} = 5\frac{3}{7}$ times.

Or, 4 is contained in $21\frac{5}{7}$, 5 times and $1\frac{5}{7}$ or $\frac{12}{7}$ remainder; $\frac{1}{4}$ of $\frac{12}{7}$ *is $\frac{3}{7}$, which added to 5* makes $5\frac{3}{7}$. Hence, etc.

16. If a man walk $18\frac{2}{3}$ miles in 4 hours, how far does he walk in 1 hour? In 3 hours? In 5 hours?

17. How many times will $16\frac{4}{5}$ gallons of cider fill a vessel that holds 3 gallons?

18. What cost 1 pound of sugar, if 6 pounds cost $\$1\frac{5}{7}$?

19. If a boy earn $\$12\frac{3}{4}$ in 10 days, how much does he earn in 1 day? In 4 days? In 5 days? In 7 days?

20. Divide $8\frac{1}{3}$ by 5; $10\frac{2}{7}$ by 7; $18\frac{1}{2}$ by 12; $4\frac{3}{4}$ by 8.

21. Divide $\frac{3}{4}$ of 21, by $\frac{2}{5}$ of 10; $\frac{7}{8}$ of 29, by $\frac{1}{9}$ of 63.

Find the value of

22. $\frac{15}{16} \div 5$.
23. $\frac{7}{10} \div 8$.
24. $1\frac{11}{12} \div 6$.
25. $4\frac{3}{8} \div 7$.
26. $9\frac{4}{9} \div 5$.
27. $13\frac{5}{7} \div 4 - 2\frac{2}{3}$.
28. $27\frac{3}{4} \div 6 + 1\frac{3}{8}$.
29. $\frac{21}{25} \div 7 \times 10$.
30. $1\frac{7}{20} \div 9 \times \frac{5}{6}$.
31. $\overline{5\frac{3}{8} \times 4} + \overline{13\frac{1}{2} \div 3}$.
32. $\frac{14}{15} \div 7 \times \overline{\frac{2}{3} \text{ of } 1\frac{1}{2}}$.
33. $\frac{7}{20} - \frac{1}{5} \times \overline{\frac{5}{8} \div 4}$.

234. PRINCIPLE.—*Dividing the numerator or multiplying the denominator divides the fraction.* (**200,** 2.)

Always divide the numerator when it is a multiple of the divisor; otherwise multiply the denominator.

WRITTEN EXERCISES.

235. 1. Divide $\frac{18}{25}$ by 6.

OPERATION.

$$\frac{18}{25} \div 6 = \frac{18 \div 6}{25} = \frac{3}{25}$$

Or, $\frac{18}{25} \div 6 = \frac{18}{25 \times 6} = \frac{3}{25}$

ANALYSIS.—First, to divide $\frac{18}{25}$ by 6, divide the numerator of the fraction by 6, which gives $\frac{3}{25}$ for the quotient. Or,

Multiply the denominator of the fraction by 6, which gives the same result.

In the first operation, the *number* of parts or of fractional units is diminished, while their *size* or value remains the same; in the second operation, the *number* of the parts remains unchanged, while their *size* is diminished.

Divide

2. $\frac{32}{47}$ by 8.
3. $\frac{125}{216}$ by 25.
4. $\frac{13}{17}$ by 18.
5. $\frac{105}{311}$ by 21.
6. $\frac{128}{1026}$ by 64.
7. $\frac{95}{114}$ by 35.

8. Divide $50\frac{2}{5}$ by 8.

OPERATION.

$$50\tfrac{2}{5} \div 8 = \frac{252}{5\times 8} = \frac{252}{40} = 6\tfrac{3}{10}$$

$$\text{Or,}\quad 8\,\underline{)\,50\tfrac{2}{5}}$$
$$6\tfrac{3}{10}$$

ANALYSIS.—Reduce the mixed number to an improper fraction and divide as in Ex. 1. Or,

Divide as in simple numbers; 8 is contained in $50\frac{2}{5}$, 6 times and a remainder of $2\frac{2}{5}$; $2\frac{2}{5}$ equal $\frac{12}{5}$, which divided by 8 gives $\frac{12}{40}$ or $\frac{3}{10}$, which added to the partial quotient 6 gives $6\frac{3}{10}$, the required quotient.

What is the quotient

9. Of $42\frac{7}{8}$ divided by 7?
10. Of $128\frac{11}{20}$ divided by 8?
11. Of $85\frac{7}{36}$ divided by 21?
12. Of $248\frac{1}{3}$ divided by 48?
13. Of $306\frac{5}{12}$ divided by 25?
14. Of $510\frac{5}{6}$ divided by 30?

15. If 20 pounds of rice cost \$$1\frac{2}{3}$, what is the cost of 1 pound?

16. The product of two numbers is $72\frac{5}{6}$, and one of them is 14; what is the other?

17. If $\frac{35}{36}$ of an acre produce 25 bushels of wheat, what part of an acre will produce 1 bushel?

18. If 12 ploughs cost \$$124\frac{3}{4}$, what is the cost of each?

19. What number multiplied by 48 produces $694\frac{2}{5}$?

20. If 54 horses cost \$$4622\frac{2}{5}$, what is the cost of each?

21. The product of two numbers is $1248\frac{7}{8}$, and one of the numbers is 32; what is the other?

22. If 11 men consume $\frac{3}{4}$ of $100\frac{3}{8}$ pounds of meat in 1 week, how much does 1 man consume in the same time?

23. What is the weight of 4 tubs of lard, if 12 tubs *weigh $528\frac{1}{4}$ pounds?*

236. When the divisor is a fractional number.

ORAL EXERCISES.

1. How many halves in 1 pound? In 4 pounds?

2. 1 is how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$?

3. What is the quotient of 1 divided by $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{8}$?

4. At \$$\frac{4}{5}$ a yard, how many yards of cloth can be bought for \$6?

ANALYSIS.—As many yards as \$$\frac{4}{5}$ is contained times in \$6. 6 is equal to $\frac{30}{5}$, and 4 fifths is contained in 30 fifths $7\frac{1}{2}$ times.

Or, \$$\frac{4}{5}$ is contained in \$1, $\frac{5}{4}$ times, and in \$6, 6 times $\frac{5}{4}$ or $\frac{30}{4}$, equal to $7\frac{1}{2}$ times. Hence, etc.

5. If a boy earn \$$\frac{3}{4}$ a day, in what time will he earn \$5?

6. At \$$\frac{5}{6}$ a pound, how much tea can be bought for \$9?

7. 1 is how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{4}{7}$? $\frac{3}{8}$? $\frac{7}{10}$? $\frac{5}{12}$?

8. 3 are how many times $\frac{2}{3}$? $\frac{3}{7}$? $\frac{5}{8}$? $\frac{7}{9}$? $\frac{4}{15}$? $\frac{7}{20}$?

9. If a horse eat $\frac{3}{7}$ of a bushel of oats in a day, in how many days will he eat 6 bushels? 8 bushels? 9 bushels?

10. Divide 12 by $\frac{7}{8}$; 16 by $\frac{2}{3}$; 25 by $\frac{1}{5}$; 14 by $\frac{4}{7}$.

11. If $\frac{5}{6}$ of a bale of hay cost \$12, what will 1 bale cost?

ANALYSIS.—Since $\frac{5}{6}$ of a bale cost \$12, $\frac{1}{6}$ of a bale will cost $\frac{1}{5}$ of \$12, or \$$\frac{12}{5}$, and 1 bale, or $\frac{6}{6}$, will cost 6 times \$$\frac{12}{5}$, or \$$\frac{72}{5}$, equal to \$$14\frac{2}{5}$. Hence, etc.

12. What will 1 ton of coal cost, if $\frac{3}{8}$ of a ton cost \$7?

13. How many times is $\frac{4}{7}$ contained in 6? In 8? In 11?

14. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

15. How many turkeys can be bought for \$9 at \$$1\frac{3}{8}$ each?

16. How many garments can be made from 15 yards of cloth, if each garment contains $2\frac{1}{2}$ yards?

How is an integer divided by a fraction?

17. How many times is $1\frac{4}{5}$ contained in 5? $2\frac{3}{4}$, in 9?

18. How many times is 2 eighths of a yard contained in 6 eighths of a yard? $\frac{3}{7}$ of a mile, in $\frac{6}{7}$ of a mile?

19. At $\$\frac{3}{16}$ each, how many pine-apples can be bought for $\$\frac{6}{16}$? For $\$\frac{9}{16}$? For $\$\frac{11}{16}$? For $\$\frac{15}{16}$?

20. How many times $\frac{4}{5}$ in $\frac{8}{5}$? $\frac{5}{12}$ in $\frac{10}{12}$? $\frac{7}{16}$ in $\frac{9}{16}$?

How is a fraction divided by a fraction when they have a common denominator?

21. At $\$\frac{2}{5}$ a pound, how much tea can be bought for $\$\frac{3}{4}$?

ANALYSIS.—As many pounds as $\$\frac{2}{5}$ is contained times in $\$\frac{3}{4}$. $\$\frac{2}{5}$ equals $\$\frac{8}{20}$, and $\$\frac{3}{4}$ equals $\$\frac{15}{20}$; 8 twentieths is contained in 15 twentieths $1\frac{7}{8}$ times.

Or, $\$\frac{2}{5}$ is contained in \$1, $\frac{5}{2}$ times, and in $\frac{3}{4}$ of \$1, $\frac{3}{4}$ of $\frac{5}{2}$ or $\frac{15}{8}$, equal to $1\frac{7}{8}$ times. Hence, etc.

22. How many pounds of honey at $\$\frac{1}{4}$ a pound, can be bought for $\$\frac{5}{8}$? For $\$\frac{4}{5}$? For $\$\frac{5}{6}$? For $\$\frac{9}{10}$?

23. Divide $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{3}{4}$ by $\frac{1}{6}$; $\frac{7}{8}$ by $\frac{3}{4}$; $\frac{6}{7}$ by $\frac{1}{3}$; $\frac{7}{12}$ by $\frac{1}{4}$.

How is a fraction divided by a fraction when they have not a common denominator.

24. In $2\frac{1}{4}$ acres of land, how many building lots of $\frac{3}{8}$ of an acre each?

Reduce mixed numbers to improper fractions, then divide as you divide one fraction by another.

25. At $\$\frac{3}{16}$ a yard, how many yards of cambric can be bought for $\$\frac{3}{4}$? For $\$\frac{7}{8}$? For $\$\frac{1}{2}$? For $\$3\frac{1}{4}$?

26. At $\$\frac{2}{3}$ a bushel, how many bushels of onions can be bought for $\$4\frac{1}{2}$? For $\$5\frac{1}{4}$? For $\$3\frac{4}{5}$?

27. If a man chop $1\frac{1}{6}$ cords of wood in a day, in how many days can he chop $5\frac{1}{4}$ ccrds? $10\frac{1}{2}$ cords? 12 cords?

28. How many times will $4\frac{3}{4}$ gallons of kerosene fill a *can that holds* $\frac{1}{2}$ *of* $\frac{2}{3}$ of 1 gallon?

29. If $\frac{2}{5}$ of a box of figs cost $\$\frac{7}{8}$, what will 1 box cost ?

Analysis.—Since 2 fifths of a box cost $\$\frac{7}{8}$, 1 fifth of a box will cost $\frac{1}{2}$ of $\$\frac{7}{8}$ or $\$\frac{7}{16}$, and 1 box or $\frac{5}{5}$ will cost 5 times $\$\frac{7}{16}$ or $\$\frac{35}{16}$ equal to $\$2\frac{3}{16}$. Hence, etc.

30. If $\frac{3}{7}$ of a yard of cloth cost $\$\frac{4}{9}$, what will 1 yard cost ?

31. What cost 1 quart of wine, if $\frac{5}{12}$ of a quart cost $\$1\frac{5}{16}$?

What is the quotient	How many times
32. Of $\frac{9}{7}$ divided by 6 ?	35. Is $\frac{3}{8}$ contained in $\frac{6}{7}$?
33. Of $2\frac{4}{5}$ divided by 7 ?	36. Is $\frac{7}{3}$ contained in $2\frac{5}{9}$?
34. Of $16\frac{2}{3}$ divided by 10 ?	37. Is $\frac{5}{6}$ contained in 9 ?

237. Principle.—*To divide by a fraction, multiply by the denominator, and divide the product by the numerator.*

WRITTEN EXERCISES.

238. 1. Divide 180 by $\frac{8}{15}$.

OPERATION.

$$180 \div \frac{8}{15} = \overline{180 \times 15} \div 8 = 337\frac{1}{2}$$

Analysis.—Multiply 180 by the denominator 15, and divide the product by the numerator 8 (Prin.), which is equivalent to multiplying 180 by the *reciprocal* of $\frac{8}{15}$, or $\frac{15}{8}$. (**197.**)

In like manner divide

2. 63 by $\frac{7}{13}$.
3. 96 by $\frac{16}{21}$.
4. 120 by $\frac{7}{12}$.
5. 276 by $\frac{25}{37}$.
6. 316 by $\frac{9}{25}$.
7. 604 by $\frac{13}{32}$.

8. If $\frac{3}{7}$ of an acre of land cost \$63, what cost 1 acre ?

9. Divide 84 by $6\frac{3}{8}$?
10. Divide 195 by $9\frac{7}{15}$.
11. Divide 1260 by $42\frac{6}{7}$.
12. Divide 2400 by $35\frac{5}{8}$.

13. Paid $\frac{1}{8}$ of \$64 for $\frac{1}{7}$ of $17\frac{1}{2}$ cords of wood. What was the cost a cord ?

14. A man gave 503 acres of land to his sons, giving them $83\frac{5}{6}$ acres apiece. How many sons had he ?

15. Divide $\frac{9}{10}$ by $\frac{3}{8}$.

OPERATION.

$$\tfrac{9}{10} \div \tfrac{3}{8} = \tfrac{36}{40} \div \tfrac{15}{40} = 2\tfrac{2}{5}$$

Or, $\frac{9}{10} \div \frac{3}{8} = \frac{9}{10} \times \frac{8}{3} = \frac{72}{30} = 2\frac{2}{5}$

ANALYSIS.—$\frac{9}{10}$ divided by $\frac{3}{8}$, is equal to 36 fortieths divided by 15 fortieths, which gives $2\frac{2}{5}$, the required quotient. Or,

Since 1 divided by $\frac{3}{8}$ is $\frac{8}{3}$, $\frac{9}{10}$ of 1 divided by $\frac{3}{8}$ is $\frac{9}{10}$ of $\frac{8}{3}$, or $\frac{72}{30}$ equal to $2\frac{2}{5}$, the same result.

It is obvious that the result is obtained by multiplying the numerator of the dividend by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor. Hence by inverting the terms of the divisor and using its *reciprocal* (**197**). the operation becomes the same as multiplying one fraction by another.

16. Divide $\frac{3}{5}$ of $\frac{5}{9}$ by $\frac{7}{9}$ of $\frac{5}{14}$.

OPERATION.

$\frac{3}{5}$ of $\frac{5}{9} = \frac{1}{3}$; $\frac{7}{9}$ of $\frac{5}{14} = \frac{5}{18}$

$\frac{1}{3} \div \frac{5}{18} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5} = 1\frac{1}{5}$

Or,

$$\begin{array}{r|l} 5 & 3 \\ \cancel{9} & \cancel{5} \\ \cancel{7} & \cancel{9} \\ \cancel{5} & \cancel{14}^{2} \\ \hline 5 & 6 \\ \cline{2-2} & 1\frac{1}{5} \end{array}$$

ANALYSIS.—The dividend reduced to its simplest form is $\frac{1}{3}$; the divisor reduced in like manner is $\frac{5}{18}$, and $\frac{1}{3}$ divided by $\frac{5}{18}$ is $1\frac{1}{5}$, the quotient required. Or,

Invert both factors of the divisor, and obtain the result by a single operation.

If the vertical line is used, the numerators of the dividend and the denominators of the divisor are written on the right.

In like manner

17. Divide $\frac{4}{7}$ by $\frac{2}{3}$.
18. Divide $\frac{9}{20}$ by $\frac{3}{5}$.
19. Divide $\frac{2}{3}$ by $\frac{27}{28}$.
20. Divide $\frac{3}{4}$ of $1\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{1}{2}$.
21. Divide $3\frac{1}{4}$ by $\frac{4}{5}$ of $2\frac{1}{2}$.
22. Divide $\frac{2}{7}$ of $2\frac{1}{3}$ by $\frac{7}{9}$ of 3.

Hence the following *general*

RULE.—*Reduce the dividend and divisor to fractions having a common denominator, then divide the numerator of the dividend by the numerator of the divisor.* Or,

I. *Reduce the integers and mixed numbers, if any, to improper fractions.*

II. *Multiply the dividend by the reciprocal of the divisor.*

Apply cancellation when practicable.

Divide

23. 56 by $1\frac{5}{9}$.
24. $\frac{128}{135}$ by 80.
25. $1\frac{14}{91}$ by $\frac{35}{52}$.
26. $\frac{3}{10}$ by $\frac{11}{24}$.
27. $45\frac{3}{4}$ by 8.
28. 92 by $5\frac{1}{6}$.
29. $16\frac{1}{2}$ by $13\frac{1}{5}$.
30. $\frac{9}{10}$ of 4 by $\frac{5}{6}$ of $3\frac{1}{4}$.
31. 44 by $\frac{2}{3}$ of $5\frac{1}{2}$ × 7.

32. What number multiplied by $\frac{3}{5}$ will produce $912\frac{15}{16}$?

33. Of what number is $52\frac{1}{2}$ the $\frac{5}{6}$ part?

34. What number multiplied by $33\frac{5}{6}$ will produce $297\frac{1}{2}$?

35. If $\frac{3}{5}$ of a farm cost $6270, what did the whole cost?

36. If 14 acres of meadow yield $32\frac{2}{3}$ tons of hay, how much do $5\frac{1}{4}$ acres produce?

37. If $7\frac{2}{5}$ yards of velvet are worth $$17\frac{1}{5}$, what is $\frac{1}{8}$ of a yard worth?

38. What will be the cost of $24\frac{1}{4}$ pounds of sugar, if $3\frac{2}{3}$ pounds cost $.33?

39. What is the value of $\dfrac{10\frac{1}{8}}{6\frac{3}{4}}$?

OPERATION.

$$\frac{10\frac{1}{8}}{6\frac{3}{4}} = \frac{\frac{81}{8}}{\frac{27}{4}} = \frac{81}{8} \div \frac{27}{4}, \text{ and}$$

$$\frac{81}{8} \div \frac{27}{4} = \frac{\overset{3}{\cancel{81}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{27}} = \frac{3}{2} = 1\frac{1}{2}$$

ANALYSIS. — This example is only another *form* for expressing division of fractions. Hence, after reducing the mixed numbers to improper fractions, treat the numerator $\frac{81}{8}$ as a *dividend*, and the denominator $\frac{27}{4}$ as a *divisor*, then proceed according to the rule for the division of fractions.

Expressions similar to the above are sometimes called *Complex Fractions*, and the process of performing the division is called *reducing a complex fraction to a simple one.*

If either the numerator or the denominator consists of one or more parts connected by + or –, the operations indicated by these signs must first be performed, then the division.

40. What is the value of $\frac{\frac{7}{8}}{\frac{3}{4}}$, or of $\frac{7}{8} \div \frac{3}{4}$?

41. What is the value of $\frac{12\frac{3}{5}}{10\frac{5}{7}}$, or of $12\frac{3}{5} \div 10\frac{5}{7}$?

42. What is the value of $\frac{117\frac{5}{9}}{18}$, or of $117\frac{5}{9} \div 18$?

43. What is the value of $\frac{\frac{3}{4} \text{ of } 2\frac{1}{3}}{\frac{9}{10}}$, or of $\overline{\frac{3}{4} \times 2\frac{1}{3}} \div \frac{9}{10}$?

44. Find the value of $\frac{\frac{2}{3} \text{ of } \frac{11}{12}}{\frac{1}{18} \text{ of } 5\frac{1}{2}}$, or of $\overline{\frac{2}{3} \times \frac{11}{12}} \div \overline{\frac{1}{18} \times 5\frac{1}{2}}$.

45. Find the value of $\frac{\frac{3}{4} - \frac{3}{5}}{\frac{1}{3} + \frac{3}{8}}$, or of $\overline{\frac{3}{4} - \frac{3}{5}} \div \overline{\frac{1}{3} + \frac{3}{8}}$.

46. If a man spend $\$4\frac{2}{5}$ a month for tobacco, in what time will he spend $\$27\frac{1}{2}$?

Find the value

47. Of $\frac{21\frac{1}{5} + 3\frac{1}{4}}{17\frac{1}{2}}$.

48. Of $\frac{\frac{3}{8} + 2\frac{5}{7}}{1\frac{6}{7} - \frac{3}{4}}$.

49. Of $\overline{\frac{5}{9} \times \frac{3}{7}} \div \overline{6\frac{1}{5} - 5\frac{4}{15}}$.

50. Of $(16\frac{3}{8} \div 18\frac{1}{4}) \times 17$.

51. Of $(9\frac{7}{9} \times 8 \times \frac{1}{15}) \div \frac{16}{17}$.

52. Of $(7\frac{4}{11} - 5\frac{1}{10}) \div (4\frac{1}{3} + 6\frac{2}{5})$.

RELATION OF NUMBERS.

239. Numbers to be compared with each other, must be so far of the same nature, that one may properly be said to be a *part* of the other.

Thus, we may compare a *day* with a *week*, since the *one* is the seventh *part* of the other; but we cannot say, that a *day* is any part of a *mile*, therefore the *former* cannot be compared with the *latter*.

240. PRINCIPLE.—*Only like numbers are so related as to be compared with each other.*

241. To find what part one number is of another.

ORAL EXERCISES.

1. What part of 5 is 3?

ANALYSIS.—Since 1 is $\frac{1}{5}$ of 5, 3 is 3 times $\frac{1}{5}$ or $\frac{3}{5}$ of 5; or it is 3 divided by 5. Hence 3 is $\frac{3}{5}$ of 5.

2. What part of 9 is 5? Of 12 is 7? Of 24 is 18?

3. 10 yards are what part of 25 yards? 8 pounds, of 20 pounds? 9 eggs, of a dozen? 10 ounces, of a pound?

4. $15 are what part of $50? $60, of $72? 9 days, of 90 days? 6 days, of a week? 7 months, of a year?

5. If an acre of land can be bought for $48, what part of an acre can be bought for $8? For $12? $16? $24?

6. What part of 3 is $\frac{5}{8}$?

ANALYSIS.—1 is $\frac{1}{3}$ of 3, and $\frac{5}{8}$ of 1 is $\frac{5}{8}$ of $\frac{1}{3}$ of 3, or $\frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$.

Or, $3 = \frac{24}{8}$; the relation of $\frac{24}{8}$ to $\frac{5}{8}$ is the same as that of their numerators 24 and 5, or $\frac{5}{24}$. Hence $\frac{5}{8}$ is $\frac{5}{24}$ of 3.

7. What part of 9 is $\frac{1}{4}$? Of 8 is $\frac{3}{10}$? Of 20 is $\frac{2}{3}$?

8. What part of 15 is $1\frac{1}{2}$? Of 18 is $2\frac{1}{3}$? Of 25 is $6\frac{1}{4}$?

9. $\frac{4}{3}$ of a month is what part of 8 months?

10. What part of $\frac{4}{5}$ is $\frac{2}{3}$?

ANALYSIS.—1 fifth is $\frac{1}{4}$ of 4 fifths, and 1, or 5 fifths, is 5 times $\frac{1}{4}$ or $\frac{5}{4}$; hence $\frac{2}{3}$ is $\frac{2}{3}$ of $\frac{5}{4}$, or $\frac{10}{12}$, equal to $\frac{5}{6}$.

Or, $\frac{4}{5} = \frac{12}{15}$, and $\frac{2}{3} = \frac{10}{15}$, and the relation of $\frac{2}{3}$ to $\frac{4}{5}$ is the same as that of 10 to 12, or $\frac{10}{12} = \frac{5}{6}$. Hence $\frac{2}{3}$ is $\frac{5}{6}$ of $\frac{4}{5}$.

11. What part of $\frac{3}{4}$ is $\frac{1}{2}$? Of $\frac{9}{16}$ is $\frac{3}{8}$? Of $\frac{5}{9}$ is $\frac{1}{6}$?

12. What part of $\frac{9}{10}$ is $\frac{3}{5}$? Of $\frac{16}{21}$ is $\frac{8}{14}$? Of $\frac{9}{11}$ is $\frac{1}{2}$?

13. What part of $3\frac{1}{2}$ is $\frac{3}{4}$? Of $4\frac{1}{3}$ is 3? Of $2\frac{4}{5}$ is $1\frac{2}{3}$?

14. What part of $7\frac{1}{2}$ is $1\frac{1}{4}$? Of $1\frac{5}{6}$ is $\frac{11}{12}$? Of $3\frac{1}{2}$ is $2\frac{1}{4}$?

15. What part of 9 miles are $\frac{3}{4}$ of 8 miles? $\frac{1}{4}$ of 10 miles?

WRITTEN EXERCISES.

242. What part of

1. 96 is 72 ?
2. 56 is $\frac{7}{8}$?
3. 120 is 90 ?
4. $\frac{13}{18}$ is $\frac{26}{42}$?
5. $\frac{15}{17}$ is $\frac{9}{34}$?
6. 6 is $\frac{45}{64}$?
7. 80 is $5\frac{1}{3}$?
8. $13\frac{3}{4}$ is $2\frac{1}{5}$?
9. 150 is $12\frac{1}{2}$?
10. $24\frac{3}{8}$ is $\frac{15}{16}$?
11. 160 is $26\frac{2}{3}$?
12. $212\frac{1}{7}$ is $42\frac{3}{7}$?

13. A man having $150, gave $25 for a robe, and $\frac{3}{5}$ of the remainder for a harness. What part of $150 had he left?

14. Bought a horse for $275, and sold him for $160. For what part of the cost was he sold ?

15. If from $18\frac{3}{4}$ yards of cloth $2\frac{7}{8}$ yards are cut, what part of the whole is taken?

16. If 15 tons of coal cost $$112\frac{1}{2}$, what part of $$112\frac{1}{2}$ will $\frac{3}{4}$ of a ton cost?

243. To find a number when a fractional part of it is given.

ORAL EXERCISES.

1. 7 is $\frac{1}{5}$ of what number.

Analysis.—7 is $\frac{1}{5}$ of 5 times 7, which is 35. Hence 7 is $\frac{1}{5}$ of 35.

2. 12 is $\frac{1}{6}$ of what number ? $\frac{1}{8}$ of what number?
3. $9\frac{1}{2}$ is $\frac{1}{7}$ of what number? $\frac{1}{4}$ of what number?
4. $7\frac{3}{8}$ is $\frac{1}{10}$ of what number ? $\frac{1}{12}$ of what number?
5. 36 is $\frac{3}{4}$ of what number?

Analysis.—Since 36 is $\frac{3}{4}$ of a certain number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 36, or 12 ; and the number is 4 times 12, or 48. Hence, etc.

6. 42 is $\frac{3}{5}$ of what number? $\frac{6}{11}$ of what number?
7. 75 is $\frac{5}{7}$ of what number ? $\frac{3}{4}$ of what number?
8. 84 is $\frac{12}{20}$ of what number? $\frac{7}{10}$ of what number ?
9. *$15\frac{3}{4}$ is $\frac{3}{7}$ of what* number? $\frac{7}{12}$ of what number?

10. $\frac{7}{8}$ is $\frac{2}{3}$ of what number? $\frac{3}{4}$ of what number?

11. $1\frac{3}{8}$ is $\frac{1}{6}$ of what number? $\frac{5}{8}$ of what number?

12. $3\frac{2}{3}$ is $\frac{5}{11}$ of what number? $\frac{5}{6}$ of what number?

13. 36 is $\frac{9}{8}$ of how many times 4?

Analysis.—36 is $\frac{9}{8}$ of 8 times $\frac{1}{9}$ of 36 which is 32, and 4 is contained in 32, 8 times. Hence 36 is $\frac{9}{8}$ of 8 times 4.

14. 28 is $\frac{7}{16}$ of how many times 8? 12? 9? 16?

15. 35 is $\frac{7}{4}$ of how many times $\frac{1}{7}$ of 28? $\frac{1}{6}$ of 30?

16. $16\frac{4}{5}$ is $\frac{7}{3}$ of how many times $\frac{1}{8}$ of 56? $\frac{1}{12}$ of 48?

17. $\frac{3}{5}$ is $\frac{5}{7}$ of how many times $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{3}{8}$?

18. $\frac{3}{8}$ of 56 is $\frac{7}{10}$ of what number?

Analysis.—$\frac{3}{8}$ of 56 is 3 times $\frac{1}{8}$ of 56, which is 21; and 21 is $\frac{7}{10}$ of 10 times $\frac{1}{7}$ of 21, which is 30. Hence $\frac{3}{8}$ of 56 is $\frac{7}{10}$ of 30.

19. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of what number? $\frac{5}{8}$ of what number?

20. $\frac{3}{8}$ of $\frac{2}{3}$ of 64 is $\frac{2}{9}$ of what number?

21. $\frac{5}{9}$ of $\frac{1}{2}$ of 72 is $\frac{1}{4}$ of $\frac{5}{8}$ of what number?

22. $\frac{7}{9}$ of 54 is $\frac{6}{5}$ of how many times 5? 7? 8? 9?

23. $\frac{3}{7}$ of $\frac{1}{3}$ of 63 is $\frac{1}{4}$ of $\frac{3}{5}$ of how many times 10? 9?

24. $\frac{4}{7}$ of 56 is $\frac{8}{9}$ of 3 times what number?

Analysis.—$\frac{4}{7}$ of 56 is 32, and 32 is $\frac{8}{9}$ of 36, and 36 is 3 times $\frac{1}{3}$ of 36, which is 12. Hence $\frac{4}{7}$ of 56 is $\frac{8}{9}$ of 3 times 12.

25. $\frac{5}{8}$ of 64 is $\frac{4}{9}$ of 9 times what number?

26. $\frac{6}{9}$ of 21 is $\frac{7}{4}$ of 8 times what number?

27. Paid \$60 for a sideboard, which was $\frac{3}{5}$ of the cost of a bookcase. What was the cost of the bookcase?

28. A scarf cost \$$1\frac{2}{3}$, which was $\frac{3}{8}$ of the cost of a vest. What was the cost of the vest?

29. Paid \$100 for a sleigh, which was $\frac{5}{6}$ of 3 times what I paid for a harness. What did I pay for the harness?

Written Exercises of this kind are included in the review examples.

REVIEW OF FRACTIONS.

ORAL EXAMPLES.

244. 1. What fraction added to $\frac{3}{7}$ will make $\frac{5}{6}$?

2. What number taken from $25\frac{2}{3}$ will leave $7\frac{3}{4}$?

3. If the sum of two fractions is $\frac{13}{20}$ and one of them is $\frac{1}{5}$, what is the other?

4. From what number must $3\frac{5}{8}$ be taken to leave $5\frac{1}{6}$?

5. A boy spends $\frac{3}{5}$ of his earnings for board, and $\frac{1}{4}$ for clothing. What part has he left?

6. The less of two numbers is $5\frac{7}{10}$, and their difference $\frac{7}{8}$. What is the greater?

7. What number divided by $\frac{4}{5}$ will give a quotient of $1\frac{2}{3}$?

8. The product of two numbers is 4, and one of them is 18. What is the other?

9. If 2 be added to both terms of the fraction $\frac{3}{5}$, will its value be increased, or diminished, and how much?

10. If 2 be added to both terms of the fraction $\frac{5}{3}$, will its value be increased, or diminished, and how much?

11. If a box of tea cost $\$21\frac{7}{8}$, what will $\frac{2}{3}$ of a box cost?

12. A man owning $\frac{7}{8}$ of a steam-mill sold $\frac{4}{5}$ of his share. What part of the whole mill does he still own?

13. A farmer sold 40 acres of land, which was $\frac{8}{15}$ of his whole farm. How many acres were there in his farm?

14. A man sold $\frac{3}{8}$ of his farm, and had 100 acres left. How many acres had he at first?

15. Bought a watch and chain for $120, the chain costing $\frac{2}{7}$ as much as the watch. What did each cost?

16. A, B, and C together own a yacht. A owns $\frac{2}{7}$ of *it, and B, $\frac{4}{9}$ of it.* What part does C own?

17. A farmer put all his grain into 4 bins : in the first he put $\frac{3}{8}$ of it, in the second $\frac{1}{4}$, in the third $\frac{1}{6}$, and in the fourth 40 bushels. How many bushels of grain had he ?

18. Bought 6 mats at \$$\frac{5}{8}$ each, and had \$5 left. How much money had I at first ?

19. How many bushels of grain can be put into 15 bags, if they hold 2$\frac{2}{3}$ bushels each ?

20. If 5 men can do a piece of work in 10$\frac{5}{6}$ days, how many days will it take one man to do the same?

21. If a man can build 6 rods of wall in 1 day, how many rods can he build in 7$\frac{3}{4}$ days ?

22. How much less than \$10 will 7 pounds of tea cost, at \$$\frac{4}{5}$ a pound ?

23. George having \$1$\frac{1}{2}$, gave $\frac{2}{3}$ of it for a knife. What part of a dollar did he give for his knife?

24. At \$12$\frac{1}{2}$ a ton, what will $\frac{5}{6}$ of a ton of hay cost?

25. Bought a cow for \$45$\frac{1}{2}$, and sold her for $\frac{9}{10}$ of what she cost. What did I lose?

26. If a man has 22$\frac{2}{3}$ bushels of clover-seed, and he sells $\frac{3}{4}$ of it, how much has he left?

27. What will 4$\frac{1}{2}$ days' wages come to at \$2$\frac{1}{4}$ a day?

28. A man spent $\frac{2}{3}$ of his money, and then found that \$15 was $\frac{3}{4}$ of what he had left. What had he at first?

29. A man paid \$30 for a cow, $\frac{4}{5}$ of the cost of which was $\frac{2}{3}$ of the cost of a horse. What did the horse cost?

30. How many pounds of tea worth \$$\frac{7}{12}$ a pound, must be given for 9 bushels of apples worth \$$\frac{4}{5}$ a bushel ?

31. How many building lots of $\frac{9}{10}$ of an acre each are contained in 1$\frac{4}{5}$ acres of land ?

32. At \$$\frac{7}{8}$ each, how many books can be bought for \$3$\frac{2}{3}$?

33. If $\frac{2}{3}$ *of a box* of figs cost \$1$\frac{7}{8}$, what will 1 box cost?

34. If $5\frac{1}{3}$ dozens of eggs cost \$$1\frac{7}{9}$, what is the cost of 1 dozen? Of $2\frac{1}{2}$ dozens? Of $3\frac{1}{6}$ dozens?

35. If $\frac{6}{7}$ of a barrel of flour cost \$8, what cost 9 barrels?

36. If 3 yards of flannel cost \$$\frac{5}{6}$, what will 8 yards cost?

37. How much tea can be bought for \$$4\frac{1}{2}$, at \$$\frac{3}{8}$ a pound?

38. If $\frac{3}{8}$ of a bushel of quinces cost \$$\frac{5}{6}$, what will 1 bushel cost? $2\frac{1}{3}$ bushels? $3\frac{3}{4}$ bushels?

39. If a gallon of syrup cost \$$\frac{3}{4}$, how many gallons can be bought for \$$\frac{9}{10}$? For \$$1\frac{5}{8}$? For \$$\frac{2}{3}$?

40. A man having \$24, gave $\frac{2}{3}$ of his money for cloverseed at \$$5\frac{1}{3}$ a bushel. How many bushels did he buy?

41. What number taken from $2\frac{1}{2}$ times $12\frac{2}{3}$ leaves $20\frac{3}{4}$?

42. A coal dealer sold $\frac{2}{7}$ of what coal he had on hand for \$90, at the rate of \$6 a ton. How many tons had he?

WRITTEN EXAMPLES.

245. 1. Change $\frac{1}{2}$ of $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{7}{9}$, to equivalent fractions whose denominator shall be 72.

2. Find the least common denominator of $\frac{3}{7}$, $\frac{2}{3}$, $\frac{1}{4}$, and $1\frac{5}{6}$.

3. The less of two numbers is $1206\frac{5}{8}$ and their difference $470\frac{4}{5}$. Find the greater number.

4. Find the value of $(3 \times \frac{7}{8} \times \frac{2}{3} \times 4\frac{2}{5}) - (3\frac{7}{8} \times \frac{2}{3} \times 4 \times \frac{2}{5})$.

5. What number multiplied by $\frac{3}{5}$ will produce $1825\frac{7}{8}$?

6. What number diminished by $\frac{3}{7}$ and $\frac{5}{9}$ of itself leaves a remainder of 144?

7. If $\frac{3}{8}$ of a farm is valued at \$$1729\frac{1}{3}$, what is the value of the whole?

8. A man gave $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money for different *objects and had \$1500* left. How much had he at first?

9. If the dividend is $\frac{7}{9}$, and the quotient $\frac{4}{39}$, what is the divisor?

10. A man owning $\frac{5}{8}$ of a cotton mill, sold $\frac{2}{3}$ of his share for $\$4560\frac{5}{6}$. What was the value of the mill?

11. A stone mason worked $23\frac{1}{3}$ days, and after paying $\frac{3}{7}$ of his earnings for board and other expenses, had $\$53\frac{1}{3}$ left. What did he receive a day?

12. Gave $6\frac{2}{3}$ pounds of butter at 36 cents a pound, for $3\frac{1}{5}$ gallons of oil. What was the oil worth a gallon?

13. A person having $271\frac{1}{2}$ acres of land, sold $\frac{1}{3}$ of it to one man, and $\frac{3}{8}$ of it to another. What was the value of the remainder at $\$57\frac{4}{5}$ an acre?

14. A man's family expenses are $\$2465\frac{1}{5}$ a year, which is $\frac{6}{7}$ of his income. What does he save?

15. If $7\frac{1}{2}$ tons of hay cost \$120, how many tons can be bought for \$78?

16. If a man travel 240 miles in $5\frac{3}{4}$ days, how far would he travel in $3\frac{1}{2}$ days?

17. A can do a certain piece of work in 8 days, and B can do it in 6 days: in what time can both together do it?

18. A, B, and C can do a piece of work in 5 days; B and C can do it in 8 days: in what time can A do it alone?

19. If $\frac{2}{3}$ of 4 acres of land cost $\$205\frac{2}{3}$, what will $\frac{3}{4}$ of 2 acres cost?

20. Bought $\frac{1}{3}$ of $25\frac{1}{3}$ yards of cloth for $\frac{1}{5}$ of $\$177\frac{1}{2}$. What was the cost per yard?

21. If $\frac{2}{3}$ of a farm is worth \$9000, what is $\frac{5}{12}$ of it worth?

22. If 8 be added to both terms of the fraction $\frac{11}{12}$, will its value be increased, or diminished, and how much?

23. If 8 be added to both terms of the fraction $\frac{10}{7}$, will its value be increased, or diminished, and how much?

24. How many bushels of oats at $⅗ a bushel, will pay for ⅝ of a barrel of flour at $9½ a barrel?

25. A man at his death left his wife $12500, which was ½ of ⅚ of his estate. At her death she left 4⁄7 of her share to her daughter. What part of the father's estate did the daughter receive?

26. Paid $1837½ for 3675 bushels of oats. What was the cost a bushel?

27. A merchant bought a cargo of flour for $2173½, and sold it for 22⁄23 of the cost, thereby losing $.25 on a barrel. How many barrels of flour did he purchase?

28. A man owning ⅘ of 156⅔ acres of land, sold ½ of ¾ of his share. How many acres did he sell, and what was the value of the remainder of his share, at $42½ an acre?

29. A horse and wagon cost $360; the horse cost 2⅓ times as much as the wagon. Find the cost of the wagon?

30. If $7½ will buy 3½ cords of wood, how many cords can be bought for $31½?

31. A dealer sold 7 barrels of apples for $32½, which was ⅝ as much as he received for all he had left, at $4 a barrel. How many barrels in all did he sell?

32. If ¾ of 9 bushels of wheat cost $13½, what will ⅞ of a bushel cost?

33. A man engaging in trade lost ⅖ of the money he invested, after which he gained $740, when he had $3500. What was his loss?

34. A boy lost ½ of his kite-string, and then added 45¾ feet, when it was just ⅘ of its original length. What was its original length?

35. There are two numbers the sum of which is 4⅕, and *their difference* ⅘. What are the numbers?

36. A man invests $\frac{1}{3}$ of his money in cotton, $\frac{1}{4}$ in sugar, $\frac{2}{13}$ in molasses, and the remainder, which is $2542, in dried fruits. What is the amount of each investment, and the total amount?

37. If $\frac{12}{25}$ of $3\frac{1}{2}$ times 1, be multiplied by $\frac{7}{8}$, the product divided by $\frac{2}{9}$, the quotient increased by $4\frac{1}{6}$, and the sum diminished by $\frac{3}{7}$ of itself, what is the remainder?

Reduce to their simplest form:

38. $\dfrac{\frac{5}{9} \text{ of } \frac{3}{7}}{6\frac{1}{5} - 5\frac{4}{15}}$.

39. $\dfrac{4\frac{3}{4} + \frac{2}{3}}{3\frac{1}{7} - 1\frac{3}{14}} \times 2$.

40. $\dfrac{\frac{1}{2} \text{ of } \frac{7}{9}}{\frac{2}{3} \text{ of } \frac{1}{4}} - \dfrac{\frac{3}{5} \text{ of } \frac{1}{6}}{\frac{5}{8} \text{ of } \frac{1}{7}}$.

41. $\dfrac{\frac{1}{2} \text{ of } 7\frac{1}{2}}{\frac{3}{8} \text{ of } 15} + \dfrac{4\frac{3}{4} \times \frac{9}{10}}{11 \times 1\frac{1}{2}}$.

Complete the following equations:

42. $\left(\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}} + \dfrac{1}{2\frac{1}{2}}\right) \div \dfrac{5\frac{1}{4}}{8\frac{3}{4}} = ?$

43. $\dfrac{3\frac{1}{9} + 2\frac{5}{6}}{3\frac{1}{9} - 2\frac{5}{6}} + \dfrac{3\frac{1}{2} \times 3\frac{1}{7}}{3\frac{1}{2} \div \frac{7}{11}} - \dfrac{9\frac{1}{3}}{7} = ?$

44. $(\overline{7\frac{1}{2} + 1\frac{17}{22}} \div \overline{8\frac{2}{3} + 3\frac{22}{33}}) - (\overline{3\frac{4}{5} + \frac{22}{45}} \div \overline{3\frac{3}{4} + 14\frac{1}{36}}) = ?$

45. $\dfrac{\dfrac{1 + \frac{1}{3}}{1 - \frac{1}{3}}}{\dfrac{1 + \frac{3}{5}}{1 - \frac{3}{5}}} = ?$

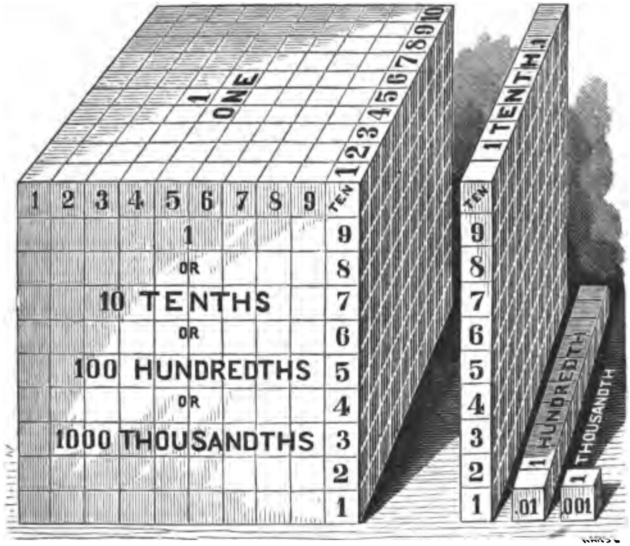
246. SYNOPSIS FOR REVIEW.

FRACTIONS.
- 1. **Equal Parts.** Principles, 1 and 2.
- 2. **Definitions.** 1. A Fraction. 2. A Fractional Unit.
- 3. **Classification.**
  - 1. Common Fractions.
  - 2. Decimal Fractions.
- 4. **Expression of Common Fractions.**
- 5. **Terms.**
  - 1. Denominator.
  - 2. Numerator.

SYNOPSIS FOR REVIEW—CONTINUED.

COMMON FRACTIONS.

- 1. **CLASSIFICATION.**
  - 1. Proper Fractions.
  - 2. Improper Fractions.
- 2. **DEFINITIONS.**
  - 1. Mixed Numbers.
  - 2. Reciprocal of a Number.
  - 3. " " Fraction.
- 3. **VALUE OF A FRACTION.**
- 4. **GENERAL PRINCIPLES**, 1, 2, 3.
- 5. **GENERAL LAW.**
- 6. **REDUCTION.**
  - **202.**
    - 1. Defs.
      - 1. *Reduction.*
      - 2. *Higher Terms.*
      - 3. *Lower Terms.*
      - 4. *Lowest Terms.*
    - 2. Principle.
    - 3. Rules, 1, 2.
  - **210.** Rule.
  - **212.** Rule.
  - **214.**
    - 1. Defs.
      - 1. *Common Denominator.*
      - 2. *Least Common Denominator.*
    - 2. Principles, 1, 2.
    - 3. Rule (1); I, II. (2.)
- 7. **ADDITION.**
  - 1. Principle.
  - 2. Rule, I, II.
- 8. **SUBTRACTION.**
  - 1. Principle.
  - 2. Rule, I, II.
- 9. **MULTIPLICATION.**
  - **227.** Principles, 1, 2. *Rule I, II* (1).
  - **230.** Principle. *Rule I, II* (2).
- 10. **DIVISION.**
  - **233.** Principle.
  - **236.** Principle.
  - *Rule I, II, III.*
- *11.* **RELATION OF NUMBERS.** Principle.
  - **241.**
  - **242.**



DECIMALS

ORAL EXERCISES.

247. 1. If a unit be divided into 10 equal parts, what is each part called? What are 2 parts? 3 parts? 4 parts?

2. What is the fractional unit?

3. If 1 *tenth* of a unit be divided into 10 equal parts, what is each part called? What are 2 parts? 4 parts? 5 parts? 7 parts? 12 parts? 25 parts?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{4}{10}$ of $\frac{1}{10}$? $\frac{6}{10}$ of $\frac{1}{10}$?

5. If a unit be divided into 100 equal parts, or each *tenth* into 10 equal parts, what are the parts called?

6. What part of 1 tenth is 1 hundredth? How many hundredths in 1 tenth?

7. If 1 *hundredth* of a unit be divided into 10 equal parts, what is each part called? What are 3 parts? What are 8 parts? 9 parts? 15 parts?

8. What is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$? $\frac{5}{10}$ of $\frac{1}{100}$?

9. If a unit be divided into 1000 equal parts, or each *hundredth* into 10 equal parts, what are the parts called? What are 12 parts? 26 parts? 42 parts?

10. What part of 1 hundredth is 1 thousandth? How many 1 thousandths is 1 hundredth?

NOTATION AND NUMERATION.

248. A ***Decimal Fraction*** is one or more of the *decimal* divisions of a unit. Thus, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$, etc., are decimal fractions.

Decimal Fractions are commonly called *Decimals.** **(16.)**

249. Decimals are like *other Fractions,* except that their denominators increase and decrease by the *uniform scale* of 10. The fractional units are, therefore, always *tenths, hundredths, thousands,* etc.

250. The ***Decimal Sign*** (**.**), called the *decimal point,* is used to distinguish a *decimal* from an *integer,* and *must always* be placed before the numerator of the decimal.

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* The terms *fraction* and *decimal* will hereafter be used to distinguish the common from the decimal *form* of expression. Thus, $\frac{3}{4}$ or $\frac{75}{100}$, and .75, are two *forms* of expressing the *same thing.* For convenience we shall call the first form a *fraction,* and the other a *decimal.*

251. The position of the decimal sign indicates the *denominator*, and determines the *value* of the decimal expression. Thus,

$\frac{7}{10}$ is expressed .7.	$\frac{126}{1000}$ is expressed .126.
$\frac{36}{100}$ " " .36.	$\frac{1425}{10000}$ " " .1425.

252. The ***Denominator*** of a decimal fraction is always 10, 100, 1000, etc., or 1 with as many ciphers annexed as there are figures in the given decimal. Thus, $.4 = \frac{4}{10}$; $.09 = \frac{9}{100}$; $.007 = \frac{7}{1000}$, etc.

253. The ***Numerator*** of a decimal fraction when expressed alone, must have as many decimal places as there are ciphers in the denominator. Thus, $\frac{8}{10} = .8$; $\frac{12}{100} = .12$; $\frac{125}{1000} = .125$, etc.

If the numerator does not contain as many figures as there are ciphers in the denominator, prefix ciphers until the number of places is equal to the number of ciphers in the denominator, and prefix the decimal point. Thus, $\frac{7}{100} = .07$; $\frac{9}{1000} = .009$, etc.

254. Decimal fractions may be written in two ways; either as *other fractions*, the denominator being expressed, or, in *decimal notation*, the denominator being omitted. Thus,

$\frac{5}{10}$, or .5	is read	5 tenths	and is $\frac{1}{10}$	of 5 units.
$\frac{5}{100}$, " .05	"	5 hundredths,	" $\frac{1}{10}$	" 5 tenths.
$\frac{5}{1000}$, " .005	"	5 thousandths,	" $\frac{1}{10}$	" 5 hundredths.

255. The value of any decimal figure is always $\frac{1}{10}$ of the value of the same figure in the next place to the left.

256. When an integer and decimal are written together, the expression is a *Mixed Number* (**195**). Thus, 7.12 and 26.134 are mixed numbers.

257. The relation of *decimals* and *integers* to each other is clearly shown by the following

TABLE.

Hund.-millions.	Ten-millions.	*Millions.*	Hund.-thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hund.-thousandths.	*Millionths.*	Ten-millionths.	Hund.-millionths.
9	8	7	6	5	4	3	2	1.	2	3	4	5	6	7	8	9
INTEGERS.									DECIMALS.							

The number is read 987 *million* 654 *thousand* 321, *and* 23 million 456 thousand 789 *hundred-millionths.*

A decimal takes the *name* of its right-hand order.

258. In decimals, as in integers, make the order of *units* the starting-point of notation and of numeration, extending the scale to the *left* of the units' place in writing *integers,* and to the *right* of the units' place in writing *decimals.*

The first order to the left of units is *tens,* and the first order to the right of units is *tenths;* the second order to the left of units is *hundreds,* and the second order to the right is *hundredths;* the third order to the left is *thousands,* and the third order to the right is *thousandths,* and so on, the *integers* on the left, and the *decimals* on the right, equally distant from the units' place, corresponding in name.

259. Hence, both in integers and in decimals, the *value* of any figure is determined by the *position* of that figure, and is always *ten times* the value of the same figure in the next *lower order,* or 1 *tenth* the value of the same figure in the *next higher* order. Hence,

260. In writing decimals, *vacant orders* must be filled *with ciphers.* (**36,** 2.)

Dictation exercises, both *oral* and *written*, should be given, until the pupil can *write* and *read* decimals with rapidity and correctness. *Oral*, thus, *Ques.* "The denominator of a fraction is 100, the numerator 7; what will express the decimal?" The prompt response should be, "*Point, naught, seven,* read, *seven-hundredths*" (.07). *Ques.* "The denominator is 1000, the numerator 35." *Ans.* "*Point, naught, three, five,* read *thirty-five thousandths*" (.035), etc.

Also the converse; thus, *Ques.* "Point, naught, eight; what will express the fraction?" *Ans.* "The numerator is *eight,* the denominator *one hundred,* and the fraction is *eight-hundredths*" ($\frac{8}{100}$). *Ques.* "Point, naught, one, five?" *Ans.* "The numerator is *fifteen,* the denominator is *one thousand,* and the fraction *fifteen-thousandths*" ($\frac{15}{1000}$), etc.

WRITTEN EXERCISES.

261. Express in the form of a fraction,

1. .12.
2. .16.
3. .138.
4. .003.
5. .2162.
6. .0056.
7. .14036.
8. .00035.

Express decimally,

9. $\frac{25}{100}$.
10. $\frac{206}{1000}$.
11. $\frac{7}{1000}$.
12. $\frac{18}{1000}$.
13. $\frac{3027}{10000}$.
14. $\frac{309}{10000}$.
15. $\frac{42}{10000}$.
16. $\frac{145}{100000}$.

262. Prefixing a cipher to a decimal multiplies the denominator by 10, and hence *divides* the decimal by 10 (**200,** 2). Thus, $.5 = \frac{5}{10}$; $.05 = \frac{5}{100}$; $.005 = \frac{5}{1000}$; or, $.5 \div 10 = .05$; $.05 \div 10 = .005$, etc.

263. Rejecting a cipher from the left of a decimal divides the denominator by 10, and hence *multiplies* the decimal by 10 (**200,** 1). Thus, $.007 = \frac{7}{1000}$; $.07 = \frac{7}{100}$; $.7 = \frac{7}{10}$; or, $.007 \times 10 = .07$; $.07 \times 10 = .7$.

264. Annexing a cipher to a decimal multiplies both numerator and denominator by 10, and hence reduces the fraction to *higher terms* (**200,** 3). Thus, $.3 = \frac{3}{10}$; $.30 = \frac{30}{100}$; $.300 = \frac{300}{1000}$.

265. Rejecting a cipher from the right of a decimal divides both numerator and denominator by 10, and hence reduces to *lower terms* (**200,** 3). Thus, $\frac{600}{1000}=.600$; $\frac{60}{100}=.60$; $\frac{6}{10}=.6$.

From the foregoing explanations are deduced the following

266. PRINCIPLES.—1. *Decimals are governed by the same laws of notation as integers.* Hence,

2. *The value of any decimal figure depends upon the place it occupies at the right of the decimal sign.* (**258.**)

3. *Every removal of a decimal figure one place to the right diminishes its value tenfold.* (**262.**)

4. *Every removal of a decimal figure one place to the left increases its value tenfold.* (**263.**)

5. *Ciphers may be annexed or rejected at the right of any decimal, without changing its value.* (**264, 265.**)

WRITTEN EXERCISES.

267. Express in figures and decimally;

1. Seventy-five thousandths. $\frac{75}{1000}=.075$.

2. Fifteen hundredths.

3. Seven thousandths.

4. Fifty-three thousandths.

5. Nine ten-thousandths.

6. 22 ten-thousandths.

7. 245 ten-thousandths.

8. 1042 hundred-thousandths.

9. 14605 millionths.

10. $\frac{128}{1000}$.

11. $7\frac{9}{1000}$.

12. $\frac{3906}{10000}$.

13. $127\frac{12}{100}$.

14. $\frac{572}{1000000}$.

15. $84\frac{308}{10000}$.

16. $\frac{13067}{100000}$.

17. $60\frac{15}{10000}$.

RULE.—I. *Write the numerator of the decimal as if an integer, writing ciphers in the place of vacant orders to give each significant figure its proper value, and place the decimal point before tenths.*

II. *Read the decimal as if an integer, and give it the name of its right-hand order.*

In like manner express decimally the following fractions and mixed numbers :

18. 596 thousandths.
19. 625 ten-thousandths.
20. 12 ten thousandths.
21. 74 millionths.
22. 105 ten-millionths.
23. 99010 billionths.

24. Four hundred thirty-seven thousand five hundred and 49 millionths.

25. Three million forty thousand and 12 ten-millionths.

26. Six hundred and 24 hundred-millionths.

27. Four hundred ninety-five million seven hundred five thousand and 43075 ten-millionths.

28. Four million seven hundred thirty-five thousand and 903624 one hundred-millionths.

29. $\frac{17}{10000}$.
30. $\frac{165}{100000}$.
31. $\frac{100354}{1000000}$.
32. $\frac{15704}{100000000}$.
33. $205\frac{65}{100}$.
34. $68\frac{36}{100000}$.
35. $705\frac{1005}{1000000}$.
36. $300\frac{10731}{100000000}$.

Copy and read the following decimals and mixed numbers :

37. .705.
38. .0023.
39. .3607.
40. .00705.
41. .400564.
42. .000256.
43. .0010275.
44. .0000407.
45. 18.0031.
46. 6.306.
47. 49.0703.
48. 10.0064.
49. 22.09042.
50. 1.10106.
51. 14.00370.
52. 70.00063.
53. .00078.
54. .3050040.
55. .0003006.
56. 42.0637.
57. 108.0094.
58. 230.40685.
59. 30.26002015.
60. 8.0401034 63.

DECIMAL CURRENCY.

268. ***Currency*** is coin, bank-bills, treasury notes, etc., employed in trade and commerce.

269. A ***Decimal Currency*** is a currency whose denominations increase and decrease by the *decimal scale.*

270. The ***Legal Currency*** of the United States is a decimal currency, and is sometimes called *Federal Money.*

TABLE.

10 mills (*m.*)	make	1 cent.	*c.* or *ct.*
10 cents	"	1 dime.	*d.*
10 dimes or 100 cents	"	1 dollar.	$.
10 dollars	"	1 eagle.	*E.*

271. Since the dollar is the *unit* of United States Money, dimes, cents, and mills are respectively *tenths, hundredths,* and *thousandths* of the unit.

272. Dollars should be written as *integers,* with the sign ($), prefixed; and dimes, cents, and mills, as *decimals,* with the decimal point at their left, or before *tenths.* Thus, 7 dollars 3 dimes 4 cents 5 mills, are written $7.345.

273. The denominations eagles and dimes are not regarded in business operations, eagles being tens of dollars, and dimes tens of cents. Thus, $34.27 is read 34 dollars 27 cents, instead of 3 eagles 4 dollars 2 dimes 7 cents.

274. Since the two places of dimes and cents, or of *tenths* and *hundredths* are appropriated to cents, when the number of cents is less than 10, write a cipher in the *place of tenths.* Thus, 9 cents are written $.09. (73.)

275. The *half-cent* may be written, either as a fraction ($\frac{1}{2}$), or as 5 mills. Thus, thirty-seven and a half cents are written $.37½, or $.375.

276. Cents are often written as fractions of a dollar. Thus, $9.28 may be also written 9\frac{28}{100}$.

277. In business transactions, if the mills in the final *result* are 5 or more than 5, they are considered a *cent*, if *less* than 5, they are not regarded. Thus, $5.197, would be called $5.20, and $3.164 would be called $3.16.

278. PRINCIPLES.—1. *Decimal currency is expressed according to the decimal system of notation.*

2. *All the operations in Decimal Currency are the same as the corresponding operations in Decimals.*

REDUCTION OF DECIMALS.

279. To reduce decimals to units of lower or higher orders.

ORAL EXERCISES.

1. How many tenths in 2 units? In 5 units?
2. How many tenths in 20 hundredths? In .40?
3. How many hundredths in 2 units? In 4 units?
4. How many hundredths in 200 thousandths?
5. How many hundredths in 5 tenths? In .6? .7? .8?
6. How many thousandths in .06? In .25? .48? .75?
7. How many hundredths in .150? In .260? In .2500?
8. In 400 thousandths how many hundredths? Tenths?
9. How many tenths of a dollar in $6? Hundredths?
10. Change 4 dollars 50 cents to cents. To mills.
11. How many dollars are 300 cents? 540 cents?
12. How many cents are 2600 mills? Dollars?

13. What is the decimal expression for 5 cents?

Ans. Sign, point, naught, five; read *five hundredths* ($.05).

14. Express decimally 7 cents; 9 cents; 15 cents.

15. Express decimally 7 mills; 5 cents 6 mills.

16. Express decimally 2 dollars 45 cents and 6 mills.

Ans. Sign, two, point, four, five, six; read, *two* and *four hundred fifty-six thousandths* dollars ($2.456).

17. What is the decimal expression for 84 cents 5 mills?

18. Change .3 to hundredths; to thousandths.

19. Change .4 and .05 to thousandths; .07 and .016.

20. Change .5, .08, and .023 to equivalent decimals, having a common denominator of 1000. Also, .14, .009, and .6. .7, .007, and .091.

21. Reduce .7, .150 and .600, to equivalent decimals, having the least common denominator. Also, .50, .250, and .1700. .43, .006, and .0214.

280. From the foregoing it appears,

1. That dollars may be reduced to cents by annexing *two* ciphers; and to mills, by annexing *three* ciphers.

Omit the sign $ and write *cts.* or *m.* after the result.

2. That cents may be reduced to mills by annexing *one* cipher.

3. That cents may be reduced to dollars by pointing off *two* figures from the right; and mills to dollars, by pointing off *three* figures from the right, and prefixing the sign ($).

4. That mills may be reduced to cents by pointing off *one* figure from the right.

5. That two or more decimals are reduced to a *common denominator* by annexing or rejecting ciphers at the right *until the decimal* places of each are equal.

WRITTEN EXERCISES.

281. Reduce

1. $85 to cents. (**280,** 1.)
2. $615 to cents.
3. $24.06 to cents.
4. $9.206 to mills.
5. $57 to mills.
6. 86 cents to mills. (**280,** 2.)
7. $.763 to mills.
8. $.47½ to mills.

Change

9. 486 cts. to dollars. (**280,** 3.)
10. 32462 cents to dollars.
11. 40327 mills to dollars.
12. 846 mills to cents.
13. 50000 mills to dollars.
14. 61040 cents to dollars.

15. Reduce .7, .05, and .304, each to hundred-thousandths. (**280,** 5.)

16. Reduce 2.5, .107, and .0008, each to ten-thousandths.

17. Change 4, 2.17, .136, and .0408 to equivalent decimals having a common denominator.

18. Reduce 9 tenths, 24 thousandths, 109 hundred-thousandths, and 47 millionths to equivalent decimals having the least common denominator. Also,

19. 100.03, 41.0034, .475, .0753, and 6.00044.

20. .84003, 120.4, 5.00031, and 15.240007.

282. To reduce a decimal to a fraction.

ORAL EXERCISES.

1. How many halves in $\frac{5}{10}$? In $\frac{50}{100}$? In $\frac{500}{1000}$?
2. How many fifths in $\frac{8}{10}$? In $\frac{40}{100}$? $\frac{60}{100}$? .6?
3. How many fourths in $\frac{25}{100}$? In .50? In .75?
4. How many twentieths in $\frac{10}{100}$? In $\frac{15}{100}$? In .20?
5. In .50 how many halves? Fourths? Tenths?

WRITTEN EXERCISES.

283. 1. Change .375 to an equivalent fraction.

OPERATION.

$.375 = \frac{375}{1000} = \frac{3}{8}$

ANALYSIS.—The numerator is 375, the denominator 1000, and the decimal expressed as a fraction is $\frac{375}{1000} = \frac{3}{8}$. Hence $.375 = \frac{3}{8}$.

Change to equivalent fractions,

2. .16.
3. .125.
4. .625.
5. \$.75.
6. \$.375.
7. \$.655.
8. .024.
9. .5625.
10. .3125.
11. \$.875.
12. .0008.
13. .9375.

RULE.—*Omit the decimal point, supply the proper denominator, and then reduce the fraction to its lowest terms.*

14. Reduce $.13\frac{1}{3}$ to an equivalent fraction.

OPERATION.—$.13\frac{1}{3} = \frac{13\frac{1}{3}}{100} = \frac{40}{300} = \frac{2}{15}$.

Reduce to fractions in their lowest terms,

15. \$$.37\frac{1}{2}$.
16. \$$.62\frac{1}{2}$.
17. \$$.08\frac{1}{3}$.
18. $.06\frac{1}{4}$.
19. $.58\frac{1}{3}$.
20. $.93\frac{3}{4}$.
21. \$$.33\frac{1}{3}$.
22. \$$.66\frac{2}{3}$.
23. \$$.16\frac{2}{3}$.
24. $.1944\frac{4}{9}$.
25. $.444\frac{4}{9}$.
26. $.0008\frac{1}{4}$.

Express by an integer and a fraction,

27. \$15.4.
28. \$36.75.
29. \$9.625.
30. \$27.375.
31. $24.26\frac{2}{3}$.
32. $84.05\frac{5}{9}$.
33. $38.41\frac{2}{3}$.
34. $104.00\frac{1}{6}$.

284. To reduce a fraction to a decimal.

ORAL EXERCISES.

1. How many tenths in $\frac{1}{2}$? How many hundredths? how many thousandths?

2. How many tenths in $\frac{1}{5}$? Hundredths in $\frac{2}{5}$? In $\frac{3}{4}$?

3. *How many* hundredths in $\frac{3}{20}$? In $\frac{4}{25}$? In $\frac{9}{50}$?

WRITTEN EXERCISES.

285. 1. Reduce $\frac{5}{8}$ to an equivalent decimal.

OPERATION.

$\frac{5}{8} = \frac{5000}{8000} = \frac{625}{1000} = .625$

ANALYSIS.—Annex the same number of ciphers to both terms of the fraction and divide the resulting terms by 8, the significant figure of the denominator, to obtain the *decimal* denominator 1000. Then change to the decimal *form*. (**253.**)

2. Reduce $\frac{2}{125}$ to an equivalent decimal.

OPERATION.

$$125\overline{)2.000}$$
$$.016$$

Or,

$\frac{2}{125} = \frac{2000}{125000} = \frac{16}{1000} = .016$

ANALYSIS.—Since $\frac{2}{125} = \frac{1}{125}$ of 2 units, and 2 units equal 2000 *thousandths*, $\frac{1}{125}$ of 2000 *thousandths* is 16 *thousandths*, or .016.

Reduce to equivalent decimals:

3. $\$\frac{3}{4}$.
4. $\$\frac{7}{8}$.
5. $\frac{14}{25}$.
6. $\frac{15}{16}$.
7. $\$\frac{4}{5}$.
8. $\$\frac{99}{200}$.
9. $\frac{3}{125}$.
10. $\frac{13}{16}$.

RULE.—I. *Annex ciphers to the numerator and divide by the denominator.*

II. *Point off as many decimal places in the result as there are ciphers annexed.*

The sign + is sometimes placed after the result to indicate that there is still a remainder. Thus, $\frac{2}{3} = .666+$, or $.666\frac{2}{3}$.

Reduce to five decimal places:

11. $\frac{5}{6}$.
12. $\frac{7}{27}$.
13. $\frac{43}{56}$.
14. $\frac{37}{150}$.

Reduce to equivalent decimals.

15. $\frac{97}{160}$.
16. $\frac{13}{256}$.
17. $\frac{1}{320}$.
18. $\frac{9}{1600}$.
19. $\frac{7}{8}$ of $\frac{3}{5}$.
20. $\frac{1}{20}$ of $\frac{16}{25}$.
21. $\frac{3}{4}$ of $\$2\frac{1}{2}$.
22. $\$2\frac{3}{4} \times \frac{3}{125}$.

Change to the decimal form:

23. $101\frac{3}{4}$.
24. $\$225\frac{5}{8}$.
25. $11\frac{1}{8}$.
26. $8.6\frac{5}{8}$.
27. $\$.93\frac{7}{16}$.
28. $\$4.0\frac{2}{25}$.
29. $\frac{31\frac{1}{3}}{8}$ of $\frac{6\frac{3}{4}}{2\frac{1}{2}}$

ADDITION.

ORAL EXERCISES.

286. 1. What is the sum of $\frac{5}{10}$ and $\frac{7}{10}$? .6 and .4?

2. What is the sum of $\frac{9}{100}$ and $\frac{13}{100}$? .11 and .15?

3. What is the sum of .12 and .20? .15 and .25?

4. Find the sum of 6 mills and 9 mills. .008 and .021.

5. What is the sum of .4 and .09? Of .04 and .009?

How many decimal figures in the sum of *tenths* and *tenths?* Of *tenths* and *hundredths?* Of *hundredths* and *thousandths?* Of *tenths* and *thousandths?* In adding several decimals, each having a different number of decimal places, how many places will there be in the sum?

287. Since decimals and integers increase and decrease uniformly by the scale of 10, decimals expressing like parts of a unit may be *added, subtracted, multiplied,* and *divided* in the same manner as integers.

The pupil should obtain and express all results in *decimal form.*

WRITTEN EXERCISES.

288. 1. Find the sum of 12.07, 326.2086, .768, and 1.9.

OPERATION.

$$\begin{array}{r} 12.0700 \\ 326.2086 \\ .7680 \\ 1.9000 \\ \hline 340.9466 \end{array}$$

ANALYSIS.—Write the numbers so that units of the same order stand in the same column. After reducing the decimals to a common denominator by annexing ciphers (280, 5), or supposing them to be annexed, add as in integers, placing the decimal point before *tenths* in the sum.

In like manner find the sum

2. Of .375, .24, .536, .0437, .50039, and .008236.

3. Of 405.327, 64.03, .84673, 121.8, and 7.00327.

4. Of $18.19, $142.095, $.964, $5.125, and $40.50.

RULE.—I. *Write the numbers so that units of the same order stand in the same column and the decimal points in the same vertical line.*

II. *Add as in addition of integers, and place the decimal point before the order of tenths in the sum.*

5. What is the sum af 37 thousandths, 54 ten-thousandths, 407 hundred-thousandths, and 12345 millionths?

6. Find the sum of 45 units, 25 tenths, 360 hundredths, 75 thousandths, 52 ten-thousandths, and 406 millionths.

Find the sum

7. Of $\$25\frac{3}{4}$, \$81.09, $\$16\frac{1}{8}$, $\$.87\frac{1}{2}$, $\$150\frac{1}{2}$, and $\$\frac{7}{8}$.

8. Of $103.60\frac{1}{2}$, $6.0\frac{7}{8}$, .37012, and $40.0034\frac{1}{8}$.

9. Of $24.6\frac{1}{2}$, $47.32\frac{7}{35}$, $5.3784\frac{1}{8}$, and $2.64878\frac{3}{4}$.

10. Of 61.843 acres, $8\frac{9}{10}$ acres, 21.04 acres, $15\frac{13}{20}$ acres, and $3\frac{4}{5}$ acres.

11. Bought a ton of coal for $\$7\frac{3}{8}$, a barrel of sugar for $\$28\frac{7}{10}$, a chest of tea for \$23.08, and a barrel of flour for $\$10.87\frac{1}{2}$. What was the cost of all?

In the reduction of each fraction, carry the decimal to at least *five* places, to insure accuracy in the *fourth*.

12. Find the sum of $\frac{3}{80}$, $\frac{2}{7}$, $\frac{43}{56}$, $\frac{7}{24}$, and $\frac{75}{436}$, in decimals, correct to the fourth place.

13. A man bought a farm for \$6736.75, which was $\$325\frac{2}{5}$ less than he sold it for. What did he sell it for?

14. How many rods of fence will enclose a field, the sides of which are respectively 34.72 rods, $48\frac{11}{25}$ rods, 152.17 rods, $95\frac{3}{8}$ rods, and $56\frac{5}{8}$ rods?

15. Paid for building a house \$3450.75, for painting the same $\$518\frac{7}{8}$, for furniture $\$1204.37\frac{1}{2}$, and for carpets $\$810\frac{4}{5}$. *What* was the cost of the whole?

SUBTRACTION.

ORAL EXERCISES.

289. 1. From $\frac{8}{10}$ take $\frac{5}{10}$. From .9 take .7.

2. From $\frac{25}{100}$ take $\frac{16}{100}$. From .36 take .12.

3. From $\frac{17}{1000}$ take $\frac{8}{1000}$. From .028 take .010.

4. From $\frac{35}{100}$ take $\frac{2}{10}$. From 45 cents take 20 cents.

5. Find the difference between $\frac{3}{5}$ and $\frac{6}{10}$. $\frac{1}{4}$ and .25.

6. Find the value of $\frac{7}{10}$ — .3 ; of .5 — $\frac{1}{4}$; .65 — .5.

7. Find the value of \$$\frac{1}{2}$ — 30 cents ; 80 cents — \$.6.

How many decimal places in the remainder, if there are *three* in the minuend and *one* in the subtrahend? If *two* in the minuend and *four* in the subtrahend? If *none* in the minuend and *three* in the subtrahend?

WRITTEN EXERCISES.

290. 1. From 3.16 subtract .2453.

OPERATION.

$$\begin{array}{r} 3.1600 \\ .2453 \\ \hline 2.9147 \end{array}$$

ANALYSIS.—Write the given numbers as in Addition, the subtrahend under the minuend, reducing the decimals to a common denominator, by annexing ciphers (**280**, 5), or supposing them to be annexed, and then subtract as in integers.

2. From 324.07 take 70.20681.

3. From \$1034 take \$500.94.

RULE.—I. *Write the subtrahend under the minuend, so that units of the same order stand in the same column.*

II. *Subtract as in subtraction of integers, and place the decimal point before the orders of tenths in the remainder.*

Find the difference, decimally, between

4. \$16$\frac{1}{2}$ and \$43$\frac{3}{16}$.

5. 1.0066 and .630482.

6. \$143$\frac{7}{8}$ and \$304.96.

7. 2 and .00345.

8. 10.0402 and 26 millionths.
9. 115 and 115 tenths.
10. 5 and 125 ten-millionths.
11. \$.875 and $\$\frac{5}{8}$.
12. $\$\frac{11}{16}$ and \$.75.
13. 7.005 and .7005.
14. $.93\frac{3}{35}$ and $1.169\frac{3}{8}$.
15. $1\frac{7}{8}$ and 1875 millionths.
16. \$200 and $\$70\frac{15}{16}$.
17. .4 and $.04\frac{1}{3}$.
18. $\frac{31}{32}$ and $\frac{49}{125}$.
19. $.1\frac{1}{18}$ and $.01\frac{5}{6}$.

20. A speculator having 7346 acres of land, sold at different times $364\frac{1}{2}$ acres, 1235.125 acres, $2700\frac{7}{8}$ acres, and 850.65 acres. How much had he left?

21. A man bought an overcoat for $\$36\frac{3}{4}$, a sack for $\$18\frac{2}{3}$, and pants for $\$8.12\frac{1}{2}$, and gave in payment one fifty, and two ten-dollar bills. What change should he receive?

Find the decimal value

22. Of $\$350 - \$38\frac{1}{2} + \$100\frac{2}{25}$.
23. Of $(2\frac{3}{4} - 1\frac{4}{5}) + (.9 - \frac{3}{16})$.
24. Of $(.37\frac{1}{2} + \frac{7}{8} + 4.2) - (2 \times .68\frac{3}{4})$.
25. Of $\$250 - (\$170\frac{20}{100} - \$14\frac{1}{4}) + \$\frac{19}{20}$.
26. Of $(\$48\frac{15}{100} + \$.97) - (\$\frac{4}{5} + \$.62\frac{1}{2} + \$\frac{5}{8})$.

MULTIPLICATION.

ORAL EXERCISES.

291. 1 What is 5 times $\frac{1}{10}$? 6 times .3? 4 times .5?

2. What is 7 times $\frac{4}{100}$? 5 times .08? 6 times .09?
3. What is $\frac{7}{10} \times 3$? $3 \times .7$? $4 \times .6$? $.5 \times 7$?
4. What is $\frac{3}{100} \times 5$? $5 \times .04$? $.05 \times 7$? $8 \times .06$?
5. What is $\frac{5}{10} \times \frac{4}{10}$? $.4 \times .3$? $.8 \times .7$? $.6 \times .9$?
6. What is $\frac{5}{100} \times \frac{5}{10}$? $.5 \times .05$? $.12 \times .6$? $.7 \times .11$?
7. What is $\frac{6}{100} \times \frac{7}{100}$? $.03 \times .07$? $.15 \times .06$?
8. What is 8 times \$.6? 7 times $\frac{9}{100}$ of a dollar?
9. What is $8 \times .5$? $8 \times .05$? $8 \times .005$? $8 \times .0005$?

How many decimal places in the product of *units* multiplied by *tenths?* *Tenths* by *tenths?* *Tenths* by *hundredths?* *Hundredths* by *hundredths?*

If there are *two* decimal figures in the multiplicand, and *two* in the multiplier, how many are there in the product? If *three* in the multiplicand and *one* in the multiplier? How many decimal places are there always in the product?

292. PRINCIPLE.—*The number of decimal places in any product is equal to the decimal places in both factors.*

WRITTEN EXERCISES.

293. 1. Multiply .64 by .8.

OPERATION.

$$\begin{array}{r} .64 \\ .8 \\ \hline .512 \end{array}$$

ANALYSIS.—Multiply as in fractions. (**232.**) Thus, $.64 \times .8 = \frac{64}{100} \times \frac{8}{10} = \frac{512}{1000} = .512$. Or,

Multiply as in integers, and since *hundredths* multiplied by *tenths* produces *thousandths*, the product must contain *three* decimal places. (PRIN.)

Multiply
2. 1.245 by .27.
3. .4056 by 35.05.

Multiply
4. 7.25 by .00012.
5. $506⅛ by .048½.

RULE.—*Multiply as in multiplication of integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places toward the *right* as *there are ciphers* in the multiplier. (**266**, 4.)

Multiply and express the product decimally:

6. $324½ by .324.
7. $175.64 by .205.
8. 5.728 by 100.
9. .6207 by 1000.

10. 5½ hundredths by 25.
11. 26000 by 26 thousandths.
12. 84 tenths by 244 hundredths.
13. 7⅝ tenths by .06¼.

Find the value

14. Of $3.126 \times .046 \times .3$.
15. Of $9\frac{3}{8} \times .07\frac{1}{2} \times 10$.
16. Of $18.75 \times 1.001 \times \frac{4}{5}$.
17. Of .25 of $\frac{9}{12} \times .04\frac{1}{2}$.
18. Of $327\frac{1}{2} \times .9 \times 4\frac{1}{4}$.
19. Of $\$8.56 \times .06\frac{1}{4} \times 100$.
20. Of $18\frac{9}{25} \times .0062\frac{1}{2} \times 1000$.
21. Of .01 of $\frac{3}{4} \times 100 \times .08\frac{1}{3}$.

22. Bought 156 pounds of cheese at $\$.12\frac{1}{2}$ a pound, 327 pounds of coffee at $\$.26\frac{3}{4}$ a pound, and 17 barrels of apples at $\$2.87\frac{1}{2}$ a barrel. What was the cost of the whole?

23. If an acre of land produce 127.25 bushels of potatoes, how many bushels will 4.375 acres produce?

What is the value

24. Of 170 barrels of apples, at $\$2\frac{5}{8}$ a barrel?
25. Of 100 cords of wood, at $4.38 a cord?
26. Of $204\frac{7}{10}$ acres of land, at $\$72\frac{3}{4}$ an acre?
27. Of $580\frac{1}{2}$ pounds of sugar, at $9\frac{1}{4}$ cents a pound?
28. Of 126 mules, at $\$97\frac{5}{8}$ each?

29. What is the cost of $3\frac{5}{8}$ bales of cloth, each bale containing 36.75 yards, at $.85 a yard?

30. A farmer sold 300 bushels of oats at $.45 a bushel, $16\frac{2}{5}$ cords of wood at $\$3\frac{7}{8}$ a cord. He received in payment 125 pounds of sugar at $\$.12\frac{1}{2}$ a pound, 36 pounds of tea at $\$\frac{7}{8}$ a pound, 6 barrels of flour at $\$8.37\frac{1}{2}$ a barrel, and the remainder in cash. How much cash did he receive?

Complete the following equations:

31. $\$450.75 - \overline{\$24\frac{1}{2} \times 3.24} + \$18\frac{3}{25} = ?$
32. $(\$200 - \$125\frac{1}{4}) \times (\frac{3}{4} + 2.5) = ?$
33. $(3.0065 \times .304) + (40\frac{3}{16} \times 10) = ?$
34. $(.00493 \times 1000) \times (1 - \frac{1}{8} + .025) = ?$
35. $(4 - .00036 + .316) - (.75 + \overline{3\frac{1}{2} - 1\frac{5}{8}}) = ?$
36. $(\overline{\frac{99}{200} \times .08\frac{1}{4}} + \overline{.03685 \times \frac{4}{5}}) \times 100 = ?$

DIVISION.

ORAL EXERCISES.

294. 1. What is $\frac{1}{3}$ of $\frac{9}{10}$? $\frac{1}{4}$ of $\frac{24}{100}$? $\frac{1}{5}$ of $\frac{45}{1000}$?

2. What is $\frac{1}{2}$ of .8? $\frac{1}{6}$ of .42? $\frac{1}{8}$ of .072?

3. Divide .8 by 4; .56 by 7; .120 by 10; .0048 by 12.

4. Divide $\frac{16}{10}$ by $\frac{4}{10}$. $\frac{45}{100}$ by $\frac{15}{100}$.

5. Divide 4.8 by 6.

ANALYSIS.—4.8 equals 48 *tenths*, and $\frac{1}{6}$ of 48 *tenths* is 8 *tenths*, or .8.

6. Divide .48 by 6; .48 by .06; .048 by .006.

7. Divide $\frac{6}{10}$ by $\frac{12}{100}$ (.6 ÷ .12); 7.5 by 2.5.

8. Multiply $\frac{8}{10}$ by $\frac{9}{10}$ $(.8 \times .9)$. Divide .72 by .9.

9. Multiply $\frac{8}{100}$ by $\frac{9}{100}$ $(.08 \times .09)$. Divide .0072 by .09.

10. The product of two factors is .096, one of which is .8; what is the other?

How many decimal places in the quotient when *tenths* are divided by *units?* *Tenths* by *tenths?* *Hundredths* by *tenths?* *Thousandths* by *hundredths?*

If there are *two* decimal figures in the divisor and *three* in the dividend, how many are there in the quotient? If *three* in the divisor and *three* in the dividend? If *none* in the divisor and *three* in the dividend? If *two* in the divisor and *none* in the dividend?

295. PRINCIPLES.—1. *The dividend must contain at least as many decimal places as the divisor before division is possible.*

2. *Since the dividend is the product of the divisor and quotient, it contains as many decimal places as both divisor and quotient.* Hence,

3. *The quotient must contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

WRITTEN EXERCISES.

296. 1. Divide .952 by .7.

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OPERATION.
.7).952
   1.36
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ANALYSIS.—Divide as in fractions. (**238.**) Thus, $.952 \div .7 = \frac{952}{1000} \div \frac{7}{10} = \frac{952}{1000} \times \frac{10}{7} = \frac{136}{100} = 1.36$. Or,

Divide as in integers, and since the dividend contains *three* decimal places, and the divisor *one*, the quotient must have *two* decimal places. (PRIN. 3.)

Divide
2. 81.6 by 3.6.
3. 675 by .15.
4. .952 by 4.76.

Divide
5. $41.25 by 33.
6. $518.70 by $14.25.
7. 345.15 by .075.

RULE.—*Divide as in division of integers, and from the right of the quotient point off as many figures as the decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient be *less* than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by *prefixing* ciphers.

2. If there be a remainder after dividing the dividend, annex ciphers, and continue the division: the ciphers annexed are decimals of the dividend.

3. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless great accuracy is required.

4. To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the *left* as *there are ciphers* in the divisor. (**266**, 3.)

8. Divide 88.476 by 1.2; by 3.6; by .01½; by 1.04.
9. Divide $56.05 by .59; $408.37½ by 27.
10. Divide $6.45 by $.45; $52 by $.65; 293.75 by 45¾.
11. Divide .0026 by .003; 3 by .450; 75 by 1000.
12. What is the quotient of 75.15208 divided by 24? by .24? by .024? by .0024? by .00024?
13. Divide $3875 by 10; by 100; by 1000; by 10000.

What is the value of

14. $645.5 \div 1000$.
15. $\$1000 \div \$.02$.
16. $\$56 \div .007$.
17. $1.904 \div 4.76$.
18. $3 \div 18\frac{3}{4}$.
19. $4.2 \div 31\frac{1}{4}$.
20. $17\frac{1}{2} \div 1000$.
21. $.73\frac{2}{5} \div 100$.
22. $\$27 \div 37\frac{1}{2}$.
23. $.001 \div 100$.
24. $100 \div .001$.
25. $\$48\frac{3}{4} \div \$\frac{2}{5}$.

26. Divide .24 by 72 ; $\frac{3}{8}$ of .24 by $\frac{3}{14}$ of .042.

27. If 64 tons of iron cost $4816, how many tons can be bought for $1730.75 ?

28. How many coats can be made from 32.4 yards of cloth, allowing 2.7 yards for each coat ?

29. At $\$287\frac{2}{5}$ each, how many horses can be bought for $4885.80 ?

30. If 125 bushels of potatoes cost $\$82\frac{1}{2}$, how many barrels, each containing $2\frac{1}{2}$ bushels, can be bought for $224.40 ?

31. If $3\frac{1}{2}$ cords of wood cost $\$11.37\frac{1}{2}$, what will $20\frac{1}{8}$ cords cost ?

32. How much sugar can be bought for $46.75, if $\frac{3}{4}$ of a hundred pounds cost $\$6\frac{3}{8}$?

33. Gave $10\frac{3}{4}$ cords of wood, worth $\$4\frac{1}{2}$ a cord, for 7.74 barrels of flour. What was the flour worth a barrel ?

34. A man sold a horse for $125, and received in payment $12\frac{1}{2}$ yards of cloth at $\$3\frac{1}{4}$ a yard, and the balance in tea at $\$.62\frac{1}{2}$. How many pounds of tea did he receive ?

Find the second member in each of the following equations :

35. Of $(1.008 \div 18 + \overline{63 \div 4000} \times 100) - \frac{4}{5} = ?$

36. Of $(714 - .714) \div (\overline{.34 - .034} \times .25 \text{ of } 6) = ?$

37. Of $(\overline{.48 \div 800} \times 10000 + \overline{6.4 \div .08}) \div .125 = ?$

38. Of $\overline{34 \times .193} + \overline{2.7 \times .4\frac{1}{5}} \div (4.81 - \frac{2}{3} \text{ of } 1.662) = ?$

39. Of $(\$262.90 \div \$.56) \times \overline{.0084 + .02\frac{7}{8}} \times 100 = ?$

40. Of $(\$1260 \times 3.49) \div \$10.47 - (\$850 \div \$6.80) = ?$

CIRCULATING DECIMALS.

ORAL EXERCISES.

297. 1. What are the prime factors of 10? Of 100?

2. Change to the decimal form $\frac{1}{2}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{3}{8}$; $\frac{7}{10}$. **(285.)**

What are the prime factors of each of the denominators of these fractions?

Are they the same as the prime factors of 10?

Can these fractions be reduced to perfect decimals?

3. Change to the decimal form, extending to four places, $\frac{1}{3}$; $\frac{2}{3}$; $\frac{7}{9}$; $\frac{8}{11}$.

Can these fractions be reduced to perfect decimals?

What are the prime factors of their denominators?

4. Change to the decimal form, extending to three places, $\frac{1}{6}$; $\frac{1}{12}$; $\frac{2}{15}$.

Can these fractions be reduced to perfect decimals?

What are the prime factors of their denominators?

How do the decimals produced by these fractions differ from the decimals produced by the fractions in examples 2 and 3?

What kind of decimals are all fractions equivalent to, that in their lowest terms have denominators containing the factors 2 or 5?

5. What figure is constantly repeated in reducing to a decimal $\frac{1}{3}$? $\frac{2}{3}$? $\frac{4}{9}$? $\frac{5}{11}$?

6. If a decimal consists of 3 repeated indefinitely, what fraction is it equal to?

7. Is there any difference between $\frac{1}{3}$ and $\frac{3}{9}$? $\frac{2}{3}$ and $\frac{6}{9}$? $\frac{6}{9}$ and $\frac{66}{99}$? $\frac{66}{99}$ and $\frac{6666}{9999}$?

8. Is there any difference between $\frac{5}{11}$ and $\frac{45}{99}$? $\frac{45}{99}$ and $\frac{4545}{9999}$?

9. If the numerator is 4444, what must be its denominator so that the fraction may equal $\frac{4}{9}$?

To change a repeating decimal number to an exact fraction, what *figures must be used* in the denominator?

DEFINITIONS AND PRINCIPLES.

298. A ***Finite Decimal*** is a perfect decimal, or one that terminates with the figures written; as, .25, .375.

299. A ***Circulating Decimal*** is a decimal in which a figure, or set of figures, is constantly repeated in the same order; as .333 +, .727272 +.

300. A ***Repetend*** is the figure or set of figures, continually repeated.

The repetend is written but once, and when it consists of a single figure a point is placed over it; when it consists of more than one figure, points are placed over the first, and over the last figure. Thus, the circulating decimal .666 +, and .297297 +, are written $.\dot{6}$, and $.\dot{2}9\dot{7}$.

301. A ***Pure Circulating Decimal*** is a decimal which commences with a repetend; as $.\dot{7}$, or $.\dot{2}7\dot{9}$.

302. A ***Mixed Circulating Decimal*** is a decimal in which the repetend is preceded by one or more decimal places called the *finite part* of the decimal; as, $.2\dot{7}$, or $.04\dot{6}48$, in which .2 or .04 is called the finite part.

303. The law for the formation of repetends will be apparent from the following:

1. $\frac{1}{9} = .1111+ \quad = .\dot{1}.$
2. $\frac{1}{99} = .01010+ \quad = .\dot{0}\dot{1}.$
3. $\frac{1}{999} = .001001+ \quad = .\dot{0}0\dot{1}.$
4. $\frac{1}{9999} = .00010001+ = .\dot{0}00\dot{1}$
5. $\frac{4}{9} = .4444+ \quad = .\dot{4}.$
6. $\frac{23}{99} = .2323+ \quad = .\dot{2}\dot{3}.$
7. $\frac{135}{999} = .135135+ \quad = .\dot{1}3\dot{5}.$
8. $\frac{1728}{9999} = .17281728+ = \dot{1}72\dot{8}$

304. Principles.—1. *Every fraction in its lowest terms, whose denominator contains no other prime factors than 2 or 5 is equivalent to a finite decimal.*

2. *Every fraction in its lowest terms, whose denominator contains other prime factors than 2 or 5 is equivalent to a circulating decimal.*

3. *Every fraction in its lowest terms, whose denominator contains* 2 *or* 5 *with other prime factors is equivalent to a mixed circulating decimal.*

4. *Every pure circulating decimal is equal to a fraction whose numerator is the repetend, and whose denominator consists of as many* 9*'s as there are places in the repetend.*

WRITTEN EXERCISES.

305. To change a fraction to a finite or to a circulating decimal.

1. Change $\frac{7}{16}$ to a finite decimal. **(285.)**

2. Change to finite decimals, $\frac{4}{5}$; $\frac{7}{8}$; $\frac{9}{25}$; $\frac{13}{16}$; $\frac{23}{40}$; $\frac{11}{125}$; and $\frac{77}{200}$. (PRIN. 1.)

3. Change to a pure circulating decimal $\frac{7}{27}$.

OPERATION.—$\frac{7}{27} = 7.000000 \div 27 = .259259+ = .\dot{2}5\dot{9}$. (PRIN. 2.)

4. Change to pure circulating decimals, $\frac{6}{7}$; $\frac{7}{9}$; $\frac{9}{11}$; $\frac{12}{37}$; $\frac{10}{21}$; and $\frac{7}{41}$.

5. Change to a mixed circulating decimal $\frac{5}{6}$.

OPERATION.—$\frac{5}{6} = 5.0000 \div 6 = .8333+ = .8\dot{3}$. (PRIN. 3.)

6. Change to mixed circulating decimals $\frac{5}{12}$; $\frac{8}{15}$; $\frac{13}{22}$; $\frac{11}{30}$; and $\frac{47}{150}$.

7. Change to finite, or to circulating decimals the following fractions: $\frac{3}{25}$; $\frac{1}{8}$; $\frac{5}{17}$; $\frac{31}{64}$; $\frac{18}{27}$; $\frac{9}{32}$; $\frac{11}{125}$; $\frac{5}{21}$; and $\frac{143}{625}$.

306. To change a pure circulating decimal to a fraction.

1. Change $.\dot{2}1\dot{6}$ to a fraction.

OPERATION.

$$\dot{2}1\dot{6} = \frac{216}{999} = \frac{8}{37}$$

ANALYSIS.—Since $.\dot{0}0\dot{1} = \frac{1}{999}$ **(303)**, $\dot{2}1\dot{6}$ is equal to $\frac{216}{999}$, which reduced to its lowest terms equals $\frac{8}{37}$. Hence $.\dot{2}1\dot{6} = \frac{8}{37}$.

Change to fractions,

2. $.\dot{4}\dot{5}$.
3. $.\dot{6}\dot{6}$.
4. $.\dot{2}9\dot{7}$.
5. $.\dot{6}7\dot{5}$.
6. $.\dot{3}2\dot{4}$.
7. $.\dot{4}15\dot{8}$.

Rule.— *Write the figures of the repetend for the numerator of a fraction, and as many 9's as there are places in the repetend for the denominator, and reduce to its lowest terms.*

In like manner change to fractions,

8. $.\dot{2}7\dot{9}$.
9. $.\dot{3}2\dot{1}$.
10. $.\dot{6}43\dot{5}$.
11. $.\dot{1}06\dot{7}$.
12. $.\dot{9}512\dot{1}$.
13. $.\dot{9}2307\dot{6}$.
14. Reduce $2.\dot{2}9\dot{7}$ to an improper fraction.
15. Reduce $12.\dot{0}8\dot{1}$ to an improper fraction.

307. To change a mixed circulating decimal to a fraction.

1. Change $.2\dot{2}\dot{7}$ to a fraction.

OPERATION.

1st. $.2\dot{2}\dot{7} = \frac{2}{10} + \frac{27}{990} = \frac{5}{22}$

Or 2d. $.2\dot{2}\dot{7} = .2\frac{27}{99} = \dfrac{2\frac{27}{99}}{10} = \dfrac{225}{990} = \dfrac{5}{22}$

$$\begin{array}{lll} & .2\dot{2}\dot{7} & \text{given decimal} \\ \text{Or 3d.} & \underline{2} & \text{finite part} \\ & .225 & \frac{225}{990} = \frac{5}{22} \end{array}$$

Analysis.—Since the repetend is not $\frac{27}{99}$, but $\frac{27}{99}$ of $\frac{1}{10} = \frac{27}{990}$, write the finite part and the repetend each as fractions and add them, the reasons for which will appear more clearly in the second solution.

Or, by an abbreviated method of reducing the fractions to a common denominator, $\overline{2 \times 99} = (\overline{2 \times 100} - 2)$; hence, $\overline{2 \times 100} + 27 - 2 = 225$ is the numerator of the equivalent common fraction.

2. Change to fractions, $.5\dot{7}$; $.04\dot{8}$; $.100\dot{4}$; $.64\dot{7}\dot{2}$.

3. *Change to* mixed numbers, $7.5\dot{4}\dot{3}$. $2.5\dot{6}\dot{4}$; $7.01\dot{2}\dot{6}$.

RULE.—*Reduce the finite part and the repetend of the given decimal each to the form of a fraction. Then add them, and reduce to lowest terms.* Or,

From the given decimal subtract the finite part for a numerator, and for a denominator write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part.

Change to fractions,

4. $.04\dot{6}4\dot{8}$.
5. $.\dot{7}85\dot{2}$.
6. $.9\dot{2}8571\dot{4}$.
7. $.35\dot{1}3\dot{5}$.
8. $.01\dot{2}\dot{6}$.
9. $5.2\dot{7}$.

To *add*, *subtract*, *multiply*, or *divide* circulating decimals, reduce them to fractions, and then perform the required operation.

For a fuller development of "Circulating Decimals" and "Continued Fractions," see "Robinson's Higher Arithmetic."

SHORT METHODS.

ORAL EXERCISES.

308. 1. What part of $\$1$ are $8\frac{1}{3}$ cents? $16\frac{2}{3}$ cents? $12\frac{1}{2}$ cents? 25 cents? 50 cents?

2. At 25 cents a pound, what cost 22 pounds of coffee?

ANALYSIS.—Since 25 cents are $\$\frac{1}{4}$, 22 pounds will cost 22 times $\$\frac{1}{4}$, or $\$\frac{22}{4}$, equal to $\$5\frac{1}{2}$, or $\$5.50$. Or,

At $\$1$ a pound, 22 pounds will cost $\$22$, and at $\$\frac{1}{4}$ a pound, $\frac{1}{4}$ of $\$22$, which is $\$5\frac{1}{2}$, or $\$5.50$.

3. What is the cost of 80 pounds of beef at $12\frac{1}{2}$ cents a pound? At $16\frac{2}{3}$ cents? At 20 cents? At 25 cents?

4. At $33\frac{1}{3}$ cents a can, what will be the cost of 25 cans of sweet corn? Of 37 cans? Of 54 cans? Of 60 cans?

5. What is the cost of 160 pounds of sugar at $6\frac{1}{4}$ cents a pound? At $8\frac{1}{3}$ cents? 10 cents? $12\frac{1}{2}$ cents?

6. How many pounds of raisins, at $16\frac{2}{3}$ cents a pound, can be bought for \$5 ?

ANALYSIS.—Since $16\frac{2}{3}$ cents are \$$\frac{1}{6}$, \$5 will buy as many pounds of raisins as \$$\frac{1}{6}$ is contained times in \$5, which are 30 times. Hence, etc.

7. At \$.50 a bushel, how many bushels of oats can be bought for \$15 ? For \$$16\frac{1}{2}$? For \$25 ?

8. At $12\frac{1}{2}$ cents a yard, how many yards of calico can I buy for 27 pounds of butter, at $33\frac{1}{3}$ cents a pound ?

9. What is the cost of 40 pairs of shoes, at \$1.25 a pair ?

ANALYSIS.—At \$1 a pair, the cost would be \$40 ; but since the price is \$1 + \$$\frac{1}{4}$, the whole cost is \$40 + $\frac{1}{4}$ of \$40, or \$50. Hence, etc.

10. At \$1.50 each, what is the cost of 48 chairs ?

11. What is the cost of 60 yards of cloth, at \$$1.12\frac{1}{2}$ a yard ? At \$$1.16\frac{2}{3}$? At \$1.25 ? At \$$1.33\frac{1}{3}$? At \$2.50 ?

12. At \$2.25 a pair, what is the cost of 12 pairs of shoes ? Of 16 pairs ? Of 18 pairs ? 20 pairs ? 25 pairs ?

DEFINITIONS.

309. ***Quantity,*** in commercial transactions, is the amount of anything bought or sold, and is estimated by the number of times it contains the *measuring unit.*

310. ***Price*** is the value in money of each *measuring unit* of any commodity.

311. ***Cost*** is the value of the *entire* quantity.

312. An ***Aliquot Part,*** or ***Even Part,*** of a number is such a part as will exactly divide that number. Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5, are aliquot parts of 10.

An *aliquot part* may be either an integer or a mixed number, *while a component factor* must be an integer.

ALIQUOT PARTS OF ONE DOLLAR.

5 cents = $\frac{1}{20}$ of \$1.
10 cents = $\frac{1}{10}$ of \$1.
20 cents = $\frac{1}{5}$ of \$1.
25 cents = $\frac{1}{4}$ of \$1.
50 cents = $\frac{1}{2}$ of \$1.

$6\frac{1}{4}$ cents = $\frac{1}{16}$ of \$1.
$8\frac{1}{3}$ cents = $\frac{1}{12}$ of \$1.
$12\frac{1}{2}$ cents = $\frac{1}{8}$ of \$1.
$16\frac{2}{3}$ cents = $\frac{1}{6}$ of \$1.
$33\frac{1}{3}$ cents = $\frac{1}{3}$ of \$1.

WRITTEN EXERCISES.

313. To find the cost of a quantity when the price is an aliquot part of one dollar.

1. What cost 951 bushels of oats, at \$.$33\frac{1}{3}$ a bushel?

OPERATION.

3) 951
   317

ANALYSIS.—At \$1 a bushel, the cost would be \$951; but since the price is $\frac{1}{3}$ of \$1 a bushel, the cost is $\frac{1}{3}$ of \$951, which is \$317. Or, the cost is $\frac{1}{3}$ as many dollars as there are bushels, and $\frac{951}{3}=317$. Hence, etc.

2. What cost 750 slates, at $33\frac{1}{3}$ cents each? At 25 cents?

3. At \$.50 each, what cost 631 shad? 1250? 1605?

RULE.—*Take such a fractional part of the given number or quantity as the price is of one dollar.*

4. What is the cost of 12 sacks of coffee, each sack containing 43 pounds, at $33\frac{1}{3}$ cents a pound?

5. A merchant sold 5 pieces of prints, each containing 28 yards, at $16\frac{2}{3}$ cents per yard, 6 pieces of sheeting, each containing 34 yards, at $8\frac{1}{3}$ cents per yard, and received in payment 41 bushels of oats at \$.50 a bushel, and the balance in money. How much money did he receive?

6. At $1.12½ a foot, what cost 324 feet of wire fence?

OPERATION.
8)324
40.5
$364.5

Analysis.—At $1 a foot, the cost would be $324; but since the cost is $1 + $⅛, the entire cost is $324 + ⅛ of $324, which is $364.50.

7. At $1.33⅓ each, what will 642 steel shovels cost?

8. What cost 320 cloth caps, at $1.20 each?

314. To find the quantity when the cost is given, and the price is an aliquot part of one dollar.

1. How many barrels, at $.50 each, can be bought for $213?

OPERATION.
$213 ÷ $½ = 426
Or, 213 × 2 = 426

Analysis.—Since $½ will pay for 1 barrel, $213 will pay for as many barrels as $½ is contained times in $213, or 426 barrels. Or, since $1 will pay for 2 barrels, $213 will pay for 213 times 2, or 426 barrels. Hence, etc.

2. How many baskets of pears can be bought for $318, at $.33⅓ each? At $.50 each?

3. How many pine-apples can be bought for $240, at 16⅔ cents each? At 20 cents? At 25 cents?

Rule.—*Divide the cost by such a fraction as will express the price as an aliquot part of one dollar.*

4. How many pounds of cheese can be bought for $350, at 6¼ cents a pound? At 8⅓? 10 cents? 12½ cents?

5. How many cocoa-nuts, at $.25, can be bought for *$150.75?*

315. To find the cost when the quantity and the price of 100, or 1000 is given.

1. What cost 564 cedar posts, at $12.25 for 100 posts?

1ST OPERATION.

$$\begin{array}{r} 12.25 \\ 564 \\ \hline 100)\underline{6909.00} \\ \$69.09 \end{array}$$

2D OPERATION.

$\$12.25 \times 5.64 = \$69.09$

ANALYSIS.—At $12.25 a post, the cost would be $12.25 × 564 = $6909. But since $12.25 is the price of 100 posts, $6909 is 100 times the cost. Hence divide by 100 (**296,** Note 4), and the result is $69.09. Or,

Since 564 is 5 and 64 hundredths (5.64), if 1 hundred cost $12.25, 5.64 will cost 5.64 times $12.25, or $69.09.

If the price is by the *thousand*, divide the product by 1000, or reduce the quantity to thousands and decimals of a thousand before multiplying.

2. What is the cost of 1684 pounds of beef, at $9.37½ a hundred pounds?

3. What cost 22840 railroad ties, at $174.55 a thousand?

4. How much is the freight on 4575 pounds of merchandise from New York to Baltimore, at $.98 for 100 pounds?

RULE.—*Multiply the price by the quantity reduced to hundreds and decimals of a hundred, or to thousands and decimals of a thousand, and point off in the product as in multiplication of decimals.*

In business transactions, the letter C is sometimes used for *hundreds*, and M for *thousands*, when the price is by the 100, or 1000.

What is the cost,

5. Of 536720 bricks, at $8.75 per M.?
6. Of 2108 feet of pine boards, at $3.12½ per C.?
7. Of 2700 pine-apples, at $16¼ per 100?
8. Of 875 feet of scantling, at $10½ per M.?
9. Of 2160 oysters, at $1.86 per 100?

10. Of 3080 fence pickets, at $5¾ per 1000?

11. Of 28642 feet of timber, at $11⅜ per M.?

12. Of 1480 pounds of maple sugar, at $12.37½ per 100?

13. What is the value of 3700 cedar rails, at $5¾ per C.?

14. What is the value of 12500 shingles, at $6⅞ per M.?

15. Find the cost of 527 feet of boards at $15½ per M., and of 972 feet of siding at $1.62½ per C.

316. To find the cost, when the quantity and the price of a ton of 2000 pounds are given.

1. What is the cost of a load of hay, weighing 2280 pounds, at $18.50 a ton?

OPERATION.

```
2)18.50
  $9.25
   2.28
 $21.09
```

ANALYSIS.—Since $18.50 is the cost of 2000, ½ of $18.50, or $9.25 is the cost of 1000 pounds; and 2280 pounds will cost 2.280 times $9.25, or $21.11.

2. At $4.75 a ton, what will a load of plaster weighing 2806 pounds cost?

3. What is the freight on 21672 pounds of iron, at $2.80 a ton?

RULE.—*Multiply one-half the price of a ton by the number of thousands and decimals of a thousand in the given quantity, as in* **315.**

4. What is the value of 150 sacks of guano, each sack containing 162½ pounds, at $51¾ a ton?

5. Find the value of 6340 pounds of Lehigh coal, at $7½ a ton, and 5080 pounds of soft coal at $6¼ a ton.

6. At $26.44 a ton, what will be the cost of 1526 *pounds of bone* dust?

LEDGER ACCOUNTS.

317. A ***Ledger*** is the *principal book* of accounts kept by business men. Into it are transferred, in a condensed form, all the items of the *Journal,* or *Day Book,* for convenient reference and preservation.

318. The *debits* (marked *Dr.*) are placed on the left, and the *credits* (marked *Cr.*) are placed on the right.

319. The ***Balance of an Account*** is the *difference* between the debit and credit sides. When this is settled, or paid, the account is said to be *balanced.*

320. Find the balance of the following Ledger Accounts:

(1.)		(2.)	
Dr.	Cr.	Dr.	Cr.
$506.76	$42.17	$2371.67	$4763.84
194.32	36.24	571.84	7061.39
173.26	8.42	90.50	8242.76
71.32	10.71	2037.69	364.96
39.46	94.30	94.46	410.31
152.60	347.16	876.54	5724.27
71.78	40.00	679.81	6317.66
320.00	12.94	4930.71	2431.27
48.50	271.19	104.13	163.55
63.41	500.50	1987.67	7063.21
56.00	11.44	142.84	451.09
410.10	81.92	522.71	200.00
72.22	10.10	3114.60	1807.36
137.89	107.09	152.91	768.72
276.44	207.16	9328.42	3024.27

ACCOUNTS AND BILLS.

321. An ***Account,*** in commercial transactions, is a record of *debits and credits.*

322. A ***Debtor*** is a person who owes another money, goods, or services.

323. A ***Creditor*** is a person to whom money, goods, or services are due from another.

324. A ***Bill*** is a written statement of money paid, of goods sold or delivered, or of services rendered. It is sometimes called an *Invoice.*

An account or bill should always state the place and the time of each transaction, the names of both the parties, the price or value of each item, and the entire cost.

325. A ***Bill is receipted*** when the words "Received Payment" are written at the bottom, and the creditor's name is signed either by himself, or by some authorized person.

326. The following abbreviations are in general use:

@	At.	Disc't	Discount.	Net	Without disc't.
% or Acc't	Account.	Do.	The same.	No.	Number.
Am't	Amount.	Doz.	Dozen.	Pay't	Payment.
Bal.	Balance.	Dr.	Debtor.	Pd.	Paid.
Bbl.	Barrel.	Exch.	Exchange.	Per	By.
Bo't.	Bought.	Fol.	Folio.	Prem.	Premium.
B. L.	Bill of Lading.	Fwd.	Forward.	Prox.	Next month.
%	Per cent.	Fr't	Freight.	Rec'd	Received.
Co.	Company.	Ins.	Insurance.	Sund's	Sundries.
Cr.	Creditor.	Inst.	This month.	Ult.	Last month.
Com.	Commission.	Int.	Interest.	Yd.	Yard.
Dft.	Draft.	Mdse.	Merchandise.	Yr.	Year.

The character @ is always followed by the *price of a unit.* Thus, 5 yd. of cloth @ $3.25, signifies, 5 yards of cloth at $3.25 *a yard; ½ lb. of tea @ $.90,* signifies ½ a pound of tea at $.90 per pound.

327. Required the footings and balances of the following bills and accounts:

(1.)

NEW YORK, May 10, 1875.

A. S. MANN & Co.,

Bought of HALSTED, HAYNES & Co.

336	yd.	Muslin, ...	@	26¢ . .	$
98½	"	Canton Flannel, . .	"	18¢ . .	
162	"	Victoria Gingham, .	"	16½¢ . .	
110	"	Cassimere, ...	"	$2.87½ .	
					$

Find the footing of this bill.

(2.)

BOSTON, June 20, 1876.

MESSRS. C. P. MEAD & SON,

Bo't of BELKNAP, BRO.

216 pairs Boys' Kip Boots, . . .	@	$2.25		
160 " " Brogans, ...	"	1.12½		
75 " Women's Fox'd Gaiters, .	"	1.25		
110 " " Enameled Boots,	"	1.37½		
6 cases Men's Calf Boots, . . .	"	75.50		
1 case Drill, 648 yd., ...	"	.14½		
36 gross Silk Buttons, ...	"	.87½		
			$	

Received Payment,

BELKNAP, BRO.

(3.)

CHARLESTON, S. C., Oct. 4, 1874.

MR. CHAS. ELLIOTT,

Bo't of WM. J. AIKIN.

8 bales, ea. 485 lb., Ordinary Tex. Cotton,	@	18¼¢		
6 " " 506 " Upland, Middlings, .	"	21½¢		
3 hhd., 215 gal., N. O. Molasses (N. Crop),	"	60¢		
			$	

Rec'd Payment by draft on N. Y.,

WM. J. AIKIN.

(4.)

CHICAGO, Sept. 10, 1876.

MESSRS. COOK & CHENEY,

Bo't of BAKER & ELLIS.

275	bbl.	Flour,	State Superfine, . .	@	$7.10		
146	"	"	Minnesota Ex., . . .	"	7.87½		
94	"	"	Wisconsin XX, . .	"	8.12½		
650	bu.	Wheat,	No. 1, Red Winter,	"	1.75		
400	"	"	Illinois, No. 1, . .	"	1.82		
368	"	Corn,	Southern White, .	"	.87½		
						$	

Rec'd Paym't by note at 4 mo.,

BAKER & ELLIS.

(5.)

SAN FRANCISCO, Jan. 1, 1875.

MR. JAMES WILDE,

To HODGE AND SON, *Dr.*

1874						
Sept.	10	To 75 lb. Sugar, ...	@	12½¢		
"	"	" 1 caddy Japan Tea, 22 lb.,	"	98¢		
Oct.	16	" 1 sack Rio Coffee, 116 lb.,	"	21¢		
"	"	" 1 " Rice, 75 lb., . . .	"	9¢		
"	"	" 1 box P. & G. Soap, 60 lb.,	"	10¢		
Nov.	1	" 25 lb. Mackerel, ...	"	8¢		
"	"	" 9 gal. Molasses, ...	"	62½¢		
"	15	" 18 lb. Soda Crackers, . .	"	9¢		
Dec.	20	" 12 " Dried Beef . . .	"	12½¢		
"	26	" 1 box S. G. Starch, 28 lb.,	"	10¾¢		
					$	

Rec'd Pay't,

HODGE & SON,

Per HENRY SCOTT.

(6.)

DETROIT, May 28, 1877.

MR. JACOB R. KENT,

To GEORGE W. PARKER, *Dr.*

Jan.	6	For Building Out-house as per contract,	$150	
"	"	" Extra Labor, ...	14	50
Mch.	20	" 15 days' work of self, @ $3½		
"	"	" 7 " " of son, " 1.50		
"	"	" 784 ft. Boards, . . " 2½ per C.		
April	16	" 2 days' work, . . . " 3.50		
"	"	" Nails, Hinges, and Sundries, . .	4	75
			$	

(7.)

Statement of Account.

ST. LOUIS, Nov. 6, 1875.

MESSRS. WOOD & COLE,

To PHELPS & DODGE, *Dr.*

April	15	To 30 tons Eng. Iron, . . @	$34.30	$	
"	"	" 12 cwt. Eng. Blister Steel, "	15.25		
June	21	" 6 doz. Hoes (Trowel Steel), "	9.78		
Aug.	10	" 30 Buckeye Plows, . . "	10.45		
Oct.	3	" 12 Cross-cut Saws, . . "	12.12½		
"	"	" 37 cwt. Bar Lead, . . "	6.90		
				$	
		Cr.			
May	25	By 22 M. feet of Boards, . @	$27.60		
July	14	" 36 M. " Plank, . "	13.37½		
"	"	" 45 M. Shingles, . . . "	3.62½		
Sept.	5	" Draft on New York, ...		$500	
"	12	" 46 C. feet Scantling, . @	1.38		
				$	
		Bal. due PHELPS & DODGE,		$	

(8.)

Account Current; Balanced by Note.

Geo. B. Damon & Co.,

In % with **Gray & Banks,**

Dr. *Cr.*

1876					1876				
Aug.	2	To 796 lb. Butter @ $.28			Nov.	3	By 27 bbl. Pears @ $9.25		
Sept.	17	“ 972 “ Cheese “ .09			“	24	“ 56 “ Apples “ 1.87		
“	24	“ 481½ “ Lard “ .12			Dec.	1	“ 70¾ bu. Corn “ .70		
Oct.	4	“ 509¾ “ Tallow “ .16			“	22	“ 31½ “ Peas “ 1.95		
“	18	“ 81 doz. Eggs “ .26			1877				
“	31	“ 15 bbl. Salt “ 2.40			Jan.	2	“ Note at 3 mo. to Bal.		
Dec.	15	“ 963 lb. Hams “ .14	—	—				—	—

Gray & Banks.

Philadelphia, Jan. 1, 1877.

REVIEW.

WRITTEN EXAMPLES.

328. What is the cost,

1. Of 7½ barrels of flour, if 4¾ barrels cost $38?
2. Of 9¼ tons of coal, if .875 of a ton cost $5.635?
3. Of 14.25 yards of cloth, if 36.48 yards cost $54.72?
4. Of 100 pounds of pork, if .93 cwt. cost $6.975?
5. Of 25.42 acres of land, if .125 of an acre cost $15⅞?
6. Of 1 ton of plaster, if 1680 pounds cost $2.856?
7. Of .8 of a pound of tea, if 1 pound cost $.62½?
8. Of 18640 feet of timber, at $6¼ per C.?
9. Of 1375 pounds of potash, at $121½ a ton?
10. Of 19600 bricks, at $9¾ per M.?
11. Of .625 of a ton of coal, at $7½ a ton?
12. *Of 35* yards of cloth, if 29 yards cost $101½?
13. *Of 1 bushel* of potatoes, if 28.8 bushels cost $9.60?

14. If 36 boxes of raisins, each containing 36 pounds, cost $\$194.40$, what is the price per pound?

15. What will be the freight on 10860 pounds of merchandise from New York to St. Louis, at $\$1.62\frac{1}{2}$ per C.?

16. How much must be paid for 1220 feet of boards, at $\$25\frac{1}{2}$ per M.; 1866 feet of scantling at $\$2.12\frac{1}{2}$ per C.; and 9525 feet of lath at $\$3\frac{1}{2}$ per M.?

17. If I pay $\$1.37$ a bushel for wheat, $\$.95$ for rye, and $\$.73$ a bushel for corn, how much, of each an equal number of bushels, can I purchase for $\$70.15$?

18. Bought $27\frac{1}{2}$ barrels of sugar for $\$453.75$, and sold it at a profit of $\$4.62\frac{1}{2}$ a barrel. At what price was it sold?

19. Three persons bought 645 tons of coal, and divided so that the first had .375 of it, the second $\frac{5}{12}$, and the third the remainder. How much did the third receive?

20. What is $814\frac{9}{20} \times 26\frac{15}{32}$ correct to 5 decimal places?

21. A person having $\$55.92$, wished to purchase an equal number of pounds of tea, coffee, and sugar; the tea at $\$.87\frac{1}{2}$, the coffee at $\$.18\frac{3}{4}$, and the sugar at $\$.10\frac{1}{4}$. How many pounds of each could he buy?

22. A dealer bought 240000 feet of lumber at $\$15.90$ per M., and retailed it out at $\$2\frac{1}{4}$ per C. What was his whole gain?

23. Three hundred seventy-five thousandths of a lot of dry goods, valued at $\$8000$, was destroyed by fire. What would a man lose who owned .15 of the entire lot?

24. Bought 150 barrels of flour @ $\$6\frac{3}{8}$, and 350 bushels of wheat @ $\$1.44$. Having sold 105 barrels of the flour @ $\$8\frac{1}{4}$, and all the wheat at $\$2.06$, at what price per barrel must the remainder of the flour be sold, to gain $\$245\frac{1}{8}$ *on the whole* investment?

25. Sold 20900 feet of timber for $339.62½, and gained thereby $78.37½. What did it cost per C.?

26. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$ to a decimal.

27. A farmer exchanged 28½ bushels of oats worth $.75 per bushel, and 453 pounds of middlings worth $1½ per hundred, for 12520 pounds of plaster. What was the plaster worth per ton?

28. A merchant tailor bought 27 pieces of broadcloth, each piece containing 19⅓ yards, at $4.31¼ a yard; and sold it so as to gain $381.87½, after deducting $9.62½ for freight. For what was the cloth sold per yard?

29. If 10½ cords of wood cost $34.12½, what cost 60⅜ cords?

30. If 1½ hundred pounds of sugar cost $12¾, how many pounds can be bought for $93½, at the same rate?

31. Paid $108 for grain, $\frac{3}{10}$ of it being barley at $.62½ per bushel, and $\frac{3}{5}$ of it wheat at $1.87½ per bushel; the balance of the money was paid for oats at $.37½ per bushel. How many bushels of grain were bought?

32. What is the value of $\left(\frac{31\frac{1}{3}}{8} + \frac{6\frac{3}{4}}{2\frac{1}{12}}\right) \div 4.23$?

33. A farmer sold to a merchant 3 loads of hay weighing respectively 1826, 1478, and 1921 pounds, at $17.60 per ton, and 281 pounds of pork at $5.25 per C. He received in exchange 31 yards of sheeting @ $.18, 11½ yards of cloth @ $4.50, and the balance in money. How much money did he receive?

34. A man expended $140.30 in the purchase of rye at $.95 a bushel, wheat at $1.37 a bushel, and corn at $.73 *a bushel, buying* the same quantity of each kind; how *many bushels* in all did he purchase?

35. A farmer had 150 acres of land, which he could have sold at one time for $100 an acre, and thereby have gained $3900; but after keeping it for a time he was obliged to sell it at a loss of $2250. What did the land cost him an acre, and for how much an acre did he sell it?

36. Bought 2500 bushels of wheat @ $1.40, and 735 bushels of oats @ $.54; I had 1470 bushels of the wheat floured, and sold it at a profit of $435.87½, and I sold 528 bushels of the oats at a loss of $30. Afterward I sold the remainder of the wheat at $1.25 per bushel, and of the oats at $.45 per bushel. Did I gain or lose, and how much?

329. SYNOPSIS FOR REVIEW.

DECIMALS.
- 1. NOTATION AND NUMERATION.
  - 1. Defs.
    - 1. *Decimal Fractions.*
    - 2. " *Sign.*
  - 2. Denominator—how composed.
  - 3. Numerator—decimal places in.
  - 4. Two ways of writing decimals.
  - 5. Value of decimal figures—how determined.
  - 6. Starting point in notation and numeration.
  - 7. Principles, 1, 2, 3, 4, 5.
  - 8. Rule, I, II.
- 2. DECIMAL CURRENCY.
  - 1. Defs.
    - 1. *Currency.*
    - 2. *Decimal Currency.*
    - 3. *Federal Money.*
  - 2. **271, 272, 274, 275, 277.**
  - 3. Principles, 1, 2.
- 3. REDUCTION.
  - 1. Art. **279.**
  - 2. **280,** 1, 2, 3, 4, 5.
  - 3. **282. 283.**
  - 4. **284. 285,** Rule, I, II.

SYNOPSIS FOR REVIEW—CONTINUED.

DECIMALS.

- 4. ADDITION. Rule, I, II.
- 5. SUBTRACTION. Rule, I, II.
- 6. MULTIPLICATION.
  - 1. Principle.
  - 2. Rule.
- 7. DIVISION.
  - 1. Principles, 1, 2, 3.
  - 2. Rule.
- 8. CIRCULATING DECIMALS.
  - 1. Definitions.
    - 1. *Finite Decimal.*
    - 2. *Circ. Decimal.*
    - 3. *Repetend.*
    - 4. *Pure Circ. Dec.*
    - 5. *Mixed Circ. Dec.*
  - 2. Principles, 1, 2, 3, 4.
  - 3. **306.** *Rule.*
  - 4. **307.** *Rule*, 1, 2.
- 9. SHORT METHODS.
  - 1. Definitions.
    - 1. *Quantity.*
    - 2. *Price.*
    - 3. *Cost.*
    - 4. *Aliquot Part.*
  - 2. **313.** *Rule.*
  - 3. **314.** *Rule.*
  - 4. **315.** *Rule.*
  - 5. **316.** *Rule.*
- 10. LEDGER ACCOUNTS.
  - 1. Definitions.
    - 1. *Ledger.*
    - 2. *Bal. of Account.*
  - 2. Position of Debits and Credits.
- 11. ACCOUNTS AND BILLS:
  - 1. Definitions.
    - 1. *Account.*
    - 2. *Debtor.*
    - 3. *Creditor.*
    - 4. *Bill.*
  - 2. Receipt of a Bill.
  - 3. Mercantile Abbreviations.

DENOMINATE NUMBERS

DEFINITIONS.

330. A ***Denominate Number*** is a concrete number, and may be either simple or compound; as, 8 quarts, 5 feet 10 inches, etc.

331. A ***Simple Denominate Number*** consists of a unit or units of but one denomination; as, 16 cents, 24 hours, 30 barrels, etc.

332. A ***Compound Denominate Number*** consists of units of two or more denominations of the same nature; as, 10 pounds 6 ounces, 5 yards 2 feet 8 inches, etc.

333. In integral numbers, and in decimals, the law of increase and decrease is by the *uniform* scale of 10; but in Compound Numbers, the scale varies.

MEASURES.

334. A ***Measure*** is a *standard unit* established by law or custom, by which *quantity*, such as extent, dimension, capacity, amount, or value, is measured or estimated.

Thus, the *standard unit* of Measures of Extension is the *yard;* of Liquid Measure, the *wine gallon;* of Dry Measure, the *Winchester bushel;* of Weight, the *Troy pound*, etc. Hence the length of a piece of cloth is ascertained by applying the *yard* measure; the capacity of a cask, by the use of the *gallon* measure; of a bin, by the use of the *bushel* measure; the weight of a body, by the *pound* weight, etc.

335. Measures may be classified into *six kinds:*

1. Extension.
2. Capacity.
3. Weight.
4. Time.
5. Angles or Arcs.
6. Money or Value.

MEASURES OF EXTENSION.

336. ***Extension*** is that which has one or more of the dimensions *length, breadth,* and *thickness.* It may be a *line,* a *surface,* or a *solid.*

337. The ***Standard Unit*** of measures of *extension,* whether linear, surface, or solid, is the *yard.*

LINEAR MEASURE.

338. ***Linear,*** or ***Long Measure,*** is used in measuring lines and distances.

339. A ***Line*** has only *one* dimension—*length.*

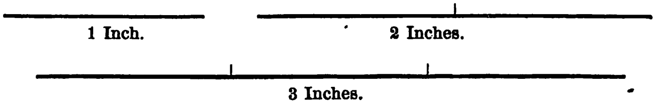


TABLE.

12	Inches (*in.*)	= 1 Foot	. .	*ft.*
3	Feet	= 1 Yard	. .	*yd.*
5½	Yards, or 16½ Feet	= 1 Rod	. .	*rd.*
320	Rods	= 1 Mile	. .	*mi.*

mi.	*rd.*	*ft.*	*in.*
1 =	320 =	5280 =	63360
	1 =	16½ =	198
		1 =	12

1. The *Inch* is generally divided into *halves, quarters, eighths, sixteenths,* and sometimes into *tenths* or *twelfths.*

2. Civil and mechanical engineers, and others, use *decimal divisions of the foot* and *inch.*

OTHER DENOMINATIONS.

3 Barley-corns, or sizes = 1 Inch. Used by shoemakers.
4 Inches = 1 Hand. " to measure the height of horses at the shoulder.
9 Inches = 1 Span. Among sailors, 8 spans = 1 fathom.
21.888 Inches = 1 Sacred Cubit.
6 Feet = 1 Fathom. Used to measure depths at sea.
120 Fathoms = 1 Cable's Length.
3 Feet = 1 Pace.
1.16 Common Miles = 1 Geog. Mi. Used to meas. distances at sea.
3 Geographic Miles = 1 League.
60 Geographic, or 69.16 Statute Miles = 1 Degree of Latitude on a Meridian, or of Longitude on the Equator.
360 Degrees = the Circumference of the Earth.

1. A *Knot* is 1 geographical or nautical mile, used to measure the speed of vessels.

2. The geographic mile is $\frac{1}{60}$ of $\frac{1}{360}$, or $\frac{1}{21600}$ of the circumference of the earth. It is a small fraction less than 1.16 common miles.

340. ***Cloth Measure*** is practically out of use. In measuring goods sold by the yard, the yard is divided into *halves, fourths, eighths,* and *sixteenths.*

At custom houses, in estimating duties, the yard is divided into *tenths* and *hundredths.*

341. ***Surveyors' Linear Measure*** is used by land surveyors in measuring roads and boundaries of land.

TABLE.

		mi.	*ch.*	*rd.*	*l.*	*in.*
7.92 Inches = 1 Link	*l.*	1 =	80 =	320 =	8000 =	63360
25 Links = 1 Rod	*rd.*		1 =	4 =	100 =	792
4 Rods = 1 Chain	*ch.*			1 =	25 =	198
80 Chains = 1 Mile	*mi.*				1 =	7.92

1. A *Gunter's Chain* is the *unit* of measure, and is 4 rods, or 66 feet long, and consists of 100 links.

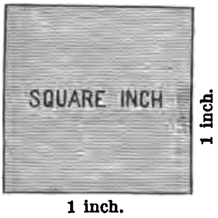
2. Engineers commonly use a chain, or measuring tape, 100 feet long.

3. *Measurements* are recorded in *chains* and *hundredths.*

SURFACE OR SQUARE MEASURE.

342. ***Surface,*** or ***Square Measure,*** is used in computing areas or surfaces.

343. A ***Surface*** has *two* dimensions—*length* and *breadth.*



344. The ***Area*** of a surface is expressed by the product of the numbers that represent these *two* dimensions.

345. A ***Square*** is a plane figure bounded by four equal sides, and having four right angles.

A *Square Inch* is a square each side of which is 1 *inch* in length.

TABLE.

144	Square Inches (*sq. in.*)	= 1 Square Foot	. . . .	*sq. ft.*
9	Square Feet	= 1 Square Yard	. . . .	*sq. yd.*
30¼	Square Yards	= 1 Square Rod or Perch	.	*sq. rd., P.*
160	Square Rods	= 1 Acre	. . . .	*A.*

sq. mi.		*A.*		*sq. rd.*		*sq. yd.*		*sq. ft.*		*sq. in.*
1	=	640	=	102400	=	3097600	=	27878400	=	4014489600

346. ***Surveyors' Square Measure*** is used by surveyors in computing the area or contents of *land.*

TABLE.

625 Square Links (*sq. l.*)	= 1 Pole	. . . .	*P.*
16 Poles	= 1 Square Chain	.	*sq. ch.*
10 Square Chains	= 1 Acre	. . . .	*A.*
640 Acres	= 1 Square Mile	.	*sq. mi.*
36 Square Miles (6 miles square)	= 1 Township	. .	*Tp.*

Tp.		*sq. mi.*		*A.*		*sq. ch.*		*P.*		*sq. l.*
1	=	*36*	=	23040	=	230400	=	3686400	=	2304000000

1. The *Acre* is the *unit* of land measure.
2. Measurements of land are commonly recorded in *square miles*, *acres*, and *hundredths* of an acre. The denomination *rood* is no longer used.*

CUBIC OR SOLID MEASURE.

347. ***Cubic,*** or ***Solid, Measure*** is used in computing the contents or volume of solids.

348. A ***Solid,*** or ***Body,*** has *three* dimensions—*length, breadth,* and *thickness.*

349. The ***Volume*** of a body is expressed by the product of the numbers that represent these dimensions.

350. A ***Cube*** is a body bounded by six equal squares, called *faces.*

The sides of these squares are called the *edges* of the cube.

A *Cubic Inch* is a cube each edge of which is 1 *inch* in length.

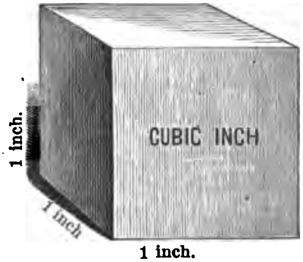

CUBIC INCH
1 inch.
1 inch
1 inch.

TABLE.

1728 Cubic In. (*cu. in.*) = 1 Cubic Ft. . *cu. ft.*
27 Cubic Ft. = 1 Cubic Yd. . *cu. yd.*

cu. yd. *cu. ft.* *cu. in.*
1 = 27 = 46656

351. ***Wood Measure*** is used to measure wood and rough stone.

TABLE.

16 Cubic Feet = 1 Cord Foot . . . . . *cd. ft.*
8 Cord Feet, or 128 Cubic Feet = 1 Cord . . . . *Cd.*
24¾ Cubic Feet = 1 Perch of Stone, or of Masonry . . *Pch.*

**For Notes* and *Applications, see "Measurements."* (   ).

ORAL EXERCISES.

352. 1. How many inches in 3 feet? In 2 ft. 6 in.?

2. How many feet in 48 in.? In 67 in.? In 75 in.?

3. In 5 yd., how many feet? In 6⅓ yd.? In 7½ yd.?

4. How many quarters in 3 yd. 2 qr.? Eighths in 5 qr.?

5. At 6 cents a quarter, what cost 3 yd. 3 qr. of cord?

6. How many yards in 96 in.? In 25 ft.? In 108 in.?

7. In 22 yd., how many rods? In 3 rd., how many ft.?

8. If a vessel sail 4 leagues an hour, how many hours will she be in sailing 75 miles?

9. How high is a horse that measures 16 hands?

10. How many fathoms deep is a body of water that requires 45 ft. of line to measure it?

11. A vessel sunk in 9½ fathoms of water: what was the depth of the water in feet?

12. What part of a foot are 9 in.? Of a yard are 12 in.?

13. How many rods is ⅛ of a mile? ¼? ⅕? ¾?

14. What part of a mile are 80 rods? 32 rd.? 64 rd.?

15. At $½ a foot, what will 6 yd. 1 ft. of lead pipe cost?

16. What part of a mile are 20 ch.? Are 60 ch.?

17. At $¾ a rod, what will it cost to dig a trench ¼ of a mile long?

18. How many square yards in 54 sq. ft.? In 84 sq. ft.?

19. In a piece of zinc 12 in. long and 9 in. wide, how many square inches?

20. Find the difference of 6 ft. square, and 6 sq. ft.?

21. In a lot 12 rd. long and 10 rd. wide, how many square rods? What part of an acre?

22. How many yards of carpeting a yard wide, will cover a floor 15 ft. long and 12 ft. wide?

23. What will it cost to pave a court 10 ft. by 15 ft., at $.50 a square foot?

24. At 20 cents a square yard, what will it cost to paint a ceiling 18 ft. by 10 ft.?

25. How many cubic feet in 2 cu. yd.? In 3 cu. yd.?

26. How many cubic inches in 1 cu. ft. 20 cu. in.?

27. What part of a cubic yard are 9 cu. ft.? Are 12 cu. ft.?

28. How many cubic feet in 3 cd. ft.? In 4 cd. ft.?

29. In ¼ of a cord, how many cord feet? Cubic feet?

30. In 2 perch of stone, how many cubic feet?

31. How many cubic inches in a 10 inch cube?

32. What is the difference between 4 cubic inches, and a 4 inch cube?

33. How many blocks, each containing 1 cu. ft., are equal to a block 6 ft. long, 5 ft. wide, and 3 ft. thick?

MEASURES OF CAPACITY.

353. ***Capacity*** signifies extent of *room* or *space.*

354. Measures of capacity are divided into two classes; Measures of Liquids and Measures of Dry Substances

355. The ***Units*** of Capacity are the *Gallon* for Liquid, and the *Bushel* for Dry Measure.

LIQUID MEASURE.

356. ***Liquid Measure*** is used in measuring *liquids.*

TABLE.

	gal.	*qt.*	*pt.*	*gi.*
4 Gills (*gi.*) = 1 Pint . . . *pt.*	1 =	4 =	8 =	32
2 Pints = 1 Quart . . *qt.*		1 =	2 =	8
4 Quarts = 1 Gallon . . *gal.*			1 =	4

In estimating the capacity of cisterns, reservoirs, etc.

31½ Gal. = 1 Barrel . . . *bbl.*
63 Gal. = 1 Hogshead . . *hhd.*

hhd.	*bbl.*	*gal.*	*qt.*	*pt.*
1 =	2 =	63 =	252 =	504
	1 =	31½ =	126 =	252

1. The barrel and hogshead are not fixed measures, but vary when used for commercial purposes.

2. The tierce, hogshead, pipe, butt, and tun are the names of casks, and do not express any fixed measures. They are usually *gauged*, and have their capacities in gallons marked on them.

357. ***Apothecaries' Fluid Measure*** is used in prescribing and in compounding *liquid* medicines.

TABLE.

60 Minims, or drops (♏) = 1 Fluidrachm . . *f*ʒ.
8 Fluidrachms = 1 Fluidounce . . *f*℥.
16 Fluidounces = 1 Pint . . . . . *O.*
8 Pints = 1 Gallon . . . . *Cong.*

Cong. 1 = *O.* 8 = *f*℥ 128 = *f*ʒ 1024 = ♏ 61440.

1. *Cong.*, for *congius*, is the Latin for *gallon*; *O.*, for *octarius*, is the Latin for *one-eighth.*

The *minim* is equivalent to a drop of water. A *pint* of water weighs a pound.

Drops are indicated in a physician's prescription by *gtt.*

The symbols, as in Apothecaries' Weight, *precede* the numbers to which they refer; thus, *O.* 3 *f.* ℥ 6, is 3 pints 6 fluid ounces.

DRY MEASURE.

358. ***Dry Measure*** is used in measuring dry articles, such as grain, fruit, roots, salt, etc.

TABLE.

2 Pints (*pt.*) = 1 Quart . . *qt.*
8 Quarts = 1 Peck . . *pk.*
4 Pecks = 1 Bushel . . *bu.*

bu.	*pk.*	*qt.*	*pt.*
1 =	4 =	32 =	64
	1 =	8 =	16
		1 =	2

For Notes and Applications, see "Measurements." (**482.**)

ORAL EXERCISES.

359. 1. How many gills in 3 pints? In 2 qt. 1 pt.?

2. How many pints in 1 gal.? In 1 gal. 2 qt. 1 pt.?

3. In 36 pints, how many quarts? How many gallons?

4. What part of a quart are 6 gi.? What part of a gallon?

5. What part of 2 gal. are 4 pints? Are 8 pt.? 2 qt.?

6. How many gills in $\frac{1}{4}$ of a quart? In $\frac{1}{8}$ of a gallon?

7. How many pints in 64 gills? How many quarts? Gallons?

8. How many fluidrachms in 5 fluidounces?

9. How many pint bottles will be required to hold 3 gal. 1 qt. of syrup? 2 gal. 3 qt.?

10. At 5 cents a pint, what will 2 gal. of milk cost?

11. If 10 gal. 2 qt. are drawn from a barrel of vinegar, how many gallons remain?

12. If a gallon of wine cost $6, what will 3 pt. cost?

13. How many barrels can be filled from 20 hogsheads?

14. At 20 cents a quart, how many gallons of molasses will $4 buy? $6? $5.60?

15. How many pints in 6 quarts? In 2 pk. 1 qt.?

16. How many quarts in 3 pk. 6 qt.? In 1 bu. 2 pk.?

17. In 96 qt., how many pecks? How many bushels?

18. What part of 5 bu. are 5 pk.? Of 1 bu. are 12 qt.?

19. How many quart boxes will 1 bu. 2 pk. 6 qt. fill?

20. At 20 cents a quart, what will $\frac{1}{2}$ bu. of plums cost?

21. At 5 cts. a pt., what is a bushel of chestnuts worth?

22. At $3.20 a bushel, how many quarts of peanuts can be bought for $2?

23. Bought $\frac{1}{2}$ bu. of chestnuts for 1\frac{1}{2}$, and sold them for 8 cents a pint. What was the gain?

MEASURES OF WEIGHT.

360. ***Weight,*** on the earth, is the measure of gravity, and varies according to the quantity of matter a body contains.

361. The ***Standard Unit*** of weight is the *Troy pound of the Mint,* and contains 5760 grains.

TROY WEIGHT.

362. ***Troy Weight*** is used in weighing gold, silver, and jewels, and in philosophical experiments.

TABLE.

24 Grains (*gr.*) = 1 Pennyweight. *pwt.*
20 Pennyweights = 1 Ounce . . . *oz.*
12 Ounces = 1 Pound . . . *lb.*

lb.	*oz.*	*pwt.*	*gr.*
1 =	12 =	240 =	5760
	1 =	20 =	480
		1 =	24

A *Carat* is a weight of about 3.2 Troy grains, and is used to weigh diamonds and precious stones.

The term *carat* is also used to express the *fineness* of gold, and means a *twenty-fourth* part. Thus, gold is said to be 18 carats fine, when it contains 18 parts of pure gold, and 6 parts of alloy, or baser metal.

APOTHECARIES' WEIGHT.

363. ***Apothecaries' Weight*** is used by physicians and apothecaries in prescribing and mixing *dry* medicines.

TABLE.

20 Grains (*gr.* xx) = 1 Scruple . . . . *sc.*, or ℈.
3 Scruples (℈iij) = 1 Dram . . . . . *dr.*, or ʒ.
8 Drams (ʒ viij) = 1 Ounce . . . . . *oz.*, or ℥.
12 Ounces (℥ xij) = 1 Pound . . . . *lb.*, or ℔.
℔ 1 = ℥ 12 = ʒ 96 = ℈288 = *gr.* 5760.

1. *Medicines* are bought and sold by Avoirdupois Weight.

2. The pound, ounce, and grain are the same as those of Troy Weight, the ounce being *differently divided.*

3. Physicians write prescriptions according to the Roman notation, using small letters, preceded by the symbols, writing j for i, when it terminates a number. Thus, 6 ounces is written, ℥ vj; 8 dr., ʒ viij; 14 sc., ℈xiv., etc.

4. ℞ is an abbreviation for *recipe*, or take; ā, aa., for equal quantities; ij. for 2; *ss.* for *semi*, or half; gr. for grain; P. for *particula*, or little part; P. æq. for equal parts; q. p., as much as you please.

AVOIRDUPOIS WEIGHT.

364. ***Avoirdupois Weight*** is used for weighing all coarse and heavy articles.

TABLE.

			T.	*cwt.*	*lb.*	*oz.*
16 Ounces (*oz.*)	=1 Pound	*lb.*	1=	20=	2000=	32000
100 Pounds	=1 Hundred-weight	*cwt.*		1=	100=	1600
20 Cwt., or 2000 lb.	=1 Ton	*T.*			1=	16

1. The *Ounce* is often divided into *halves*, *quarters*, etc.

2. The *long*, or *gross ton*, *hundred-weight*, and *quarter* were formerly in common use; but they are now seldom used, except in estimating duties at the U. S. Custom Houses, and in weighing a few of the coarser articles, such as coal at the mines, etc.

LONG TON TABLE.

			T.	*cwt.*	*qr.*	*lb.*	*oz.*
16 Ounces	= 1 Pound .	*lb.*	1=	20=	80=	2240=	35840
28 Pounds	= 1 Quarter .	*qr.*		1=	4=	112=	1792
4 Quarters	= 1 Hund. .	*cwt.*			1=	28=	448
20 Cwt., or 2240 lb.	= 1 Ton . .	*T.*				1=	16

3. Both *custom* and the *law* of most of the States make 100 pounds a *hundred-weight*.

365. The following denominations are also in use:

100 Pounds of	Grain or Flour	make	1 Cental.
100 "	Dry Fish	"	1 Quintal.
100 "	Nails	"	1 Keg.
196 "	Flour	"	1 Barrel.
200 "	Pork or Beef	"	1 Barrel.
280 "	Salt at N. Y. S. works	"	1 Barrel.
240 "	Lime	"	1 Cask.

366. The weight of the bushel of certain grains and roots has been fixed by statute in many of the States; and these statute weights must govern in buying and selling, unless specific agreements to the contrary are made.

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL,

As prescribed by statute in the several States named.

COMMODITIES.	*California.*	*Connecticut.*	*Delaware.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Maine.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*N. Hampshire.*	*New Jersey.*	*New York.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Rhode Island.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Barley	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47		46	45	48
Beans				60	60	60	60						60			60							
Blue Grass Seed				14	14	14	14						14										
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48		46	42	42
Castor Beans				46	46	46							46										
Clover Seed				60	60	60	60				60	60	60		64	60	60	60				60	60
Dried Apples				24	25	24					28	28	24					28				28	28
Dried Peaches				33	33	33					28	28	33					28				28	28
Flax Seed				55	56	56	56						56		55	55	56						56
Hemp Seed				44	44	44	44						44										
Indian Corn	52	56	56	52	56	50	56	56		56	56	56	52		56	58	56	56	56		56	56	56
Indian Corn in ear				70	68	68																	
Indian Corn Meal				48	50		50		50	50										50			
Mineral Coal				80	70	80							80										
Oats	32	28		32	32	35	33⅓	32	30	30	32	32	35	30	30	32	32	34	32		32	36	32
Onions				57	48	57	57			52			57							50		50	
Potatoes		60		60	60	60	60		60				60	60	60	60		60		60	60	60	60
Rye	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56		56	56	56
Rye Meal									50	50										50			
Salt					50	50	50						50			56							
Timothy Seed				45	45	45	45						45			44							46
Wheat	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60		60	60	60

1. In Pennsylvania 80 lb. coarse, 70 lb. ground, or 62 lb. fine salt make 1 bushel; and in Illinois, 50 lb. common, or 55 lb. fine salt *make 1 bushel.*

2. In Maine 64 lb. of ruta-baga turnips, or of beets make 1 bushel.

ORAL EXERCISES.

367. 1. How many grains in 3 pwt.? In ℈ 5?

2. How many ounces in 60 pwt.? In 100 pwt.? 120 pwt.?

3. How many ounces in 5 lb.? In 3 lb. 10 oz.? 4½ lb.?

4. How many ounces in 40 drams? In 64 dr.? 120 dr.?

5. How many pounds in 36 oz.? In 70 oz.? 110 oz.?

6. How many scruples in 10 drams? In 80 grains?

7. What will a gold chain, weighing 1 oz. 12 pwt., cost at $1 a pennyweight?

8. What part of a pound Troy are 4 oz.? 6 oz.? 8 oz.?

9. How many parts of pure gold in a ring 16 carats fine?

10. How many powders of 8 grains each, can be made from half an ounce of medicine?

11. How many tablespoons, each weighing 2 oz., can be made from 2 lb. 10 oz. of silver?

12. How many pills of gr. 5 each can be made from ℥ 1 ℈ 2 of calomel?

13. What is the value of a gold bracelet weighing 3 oz. 15 pwt., at $20 an ounce?

14. How many ounces in 4 lb. Avoir.? In 5 lb. 6 oz.?

15. How many pounds in 7 cwt.? In 8½ cwt.?

16. How many cwt. in 600 lb.? In 350 lb.? In 875 lb.?

17. In 3 T., how many hundred-weight? How many lb.?

18. What part of a cwt. are 25 lb.? 50 lb.? 75 lb.?

19. How many cwt. in ¼ of a ton? In ⅕ of a ton?

20. How many tons are 50 cwt.? 80 cwt.? 95 cwt.?

21. What will a ton of hay cost, at 1 cent a pound?

22. At 8 cents an ounce, what will 2½ lb. of licorice cost?

23. What will ¾ lb. of candy cost, at 3 cents an oz.?

24. At $2 a bushel, what must be paid for 2 bags of wheat, each containing 120 lb.?

368. SYNOPSIS FOR REVIEW.

DENOMINATE NUMBERS.

- 1. DEFINITIONS.
  - 1. Denominate Number. 2. Simple Number. 3. Comp. Denom. Number.
- 2. MEASURES.
  - 1. DEFINITION OF MEASURE.
  - 2. CLASSIFICATION.
  - 3. MEASURES OF EXTENSION.
    - 1. LINEAR MEASURE.
      - 1. For what used. 2. Table.
      - 3. Other Denominations.
      - 4. Surveyors' Linear Meas.
    - 2. SQUARE MEASURE.
      - 1. For what used.
      - 2. Defs.
        - 1. *Surface.*
        - 2. *Area.*
        - 3. *Square.*
      - 3. Table.
      - 4. Surveyors' Square Meas.
    - 3. CUBIC MEASURE.
      - 1. For what used.
      - 2. Defs.
        - 1. *Solid.* 2. *Volume.* 3. *Cube.*
      - 3. Table.
    - 4. WOOD MEASURE.
      - 1. For what used.
      - 2. Table.
  - 4. MEAS. OF CAPAC'Y.
    - 1. DEFINITION OF CAPACITY.
    - 2. UNITS OF CAPACITY.
    - 3. LIQUID MEASURE.
      - 1. For what used.
      - 2. Table.
      - 3. Apoth. Fluid Measure.
    - 4. DRY MEASURE.
      - 1. For what used.
      - 2. Table.
  - 5. MEAS. OF W'GHT.
    - 1. DEFINITION OF WEIGHT.
    - 2. STANDARD UNIT.
    - 3. TROY WEIGHT.
      - 1. For what used.
      - 2. Table
    - 4. APOTHECARIES WEIGHT.
      - 1. For what used.
      - 2. Table.
    - 5. AVOIRDUPOIS WEIGHT.
      - 1. For what used.
      - 2. Table.

MEASURES OF TIME.

369. ***Time*** is a measured portion of duration.

370. The ***Unit*** of measure is the *mean solar day.*

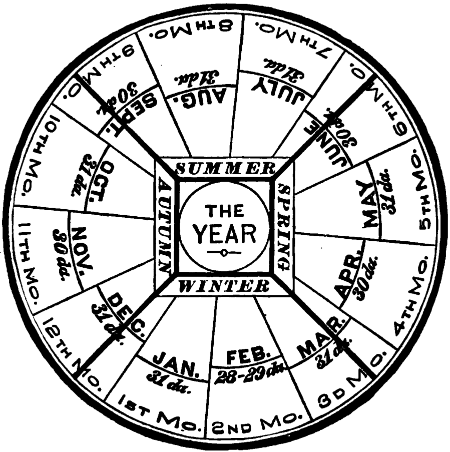
TABLE.

60 Seconds (*sec.*)	= 1 Minute	*min.*
60 Minutes	= 1 Hour	*hr.*
24 Hours	= 1 Day	*da.*
7 Days	= 1 Week	*wk.*
365 Days, or 12 Calendar Months	= 1 Common Year . .	*yr.*
366 Days	= 1 Leap Year . . .	*yr.*
100 Years	= 1 Century	*C.*

yr.	*mo.*	*da.*	*hr.*	*min.*	*sec.*
1 =	12 =	365 =	8760 =	525600 =	31536000
		366 =	8784 =	527040 =	31622400

In most business transactions 30 days are considered *a month,* and 12 months a *year.* Four weeks are sometimes called a *lunar month.*

The calendar year is divided as shown in the diagram :



365 or 366 days.

1. The *Solar Day* is the interval of time between two successive passages of the sun across the meridian of any place.

2. The *Mean Solar Day* is the mean or average length of all the solar days in the year.

3. The *Civil Day,* used for business purposes and which corresponds with the mean solar day, begins and ends at 12 o'clock, midnight. A.M. denotes the time before noon; M., at noon; and P.M., afternoon.

4. The *Solar Year* is exactly 365 da. 5 hr. 48 min. 49.7 sec.

5. The *Common Year* consists of 365 da. for 3 successive years, every *fourth* year containing 366 da., one day being added for the excess of the solar year over 365 da. This day is added to the month of February, which then has 29 da., and the year is called *Leap-year.*

371. The following rule for leap year will render the calendar correct to within 1 day, for a period of 4000 years:

I. *Every year exactly divisible by* 4 *is a leap year, the centennial years excepted; the other years are common years.*

Thus, 1876 is a leap year, but 1877 is a common year.

II. *Every centennial year exactly divisible by* 400 *is a leap year; the other centennial years are common years.*

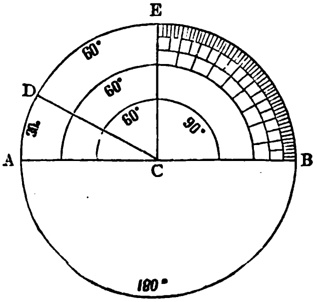
Thus, the year 2000 is a L. year, but 1800 and 1900 are com. years.

CIRCULAR MEASURE.

372. ***Circular,*** or ***Angular, Measure*** is used in measuring angles and arcs of circles, in determining latitude and longitude, the location of places and vessels, etc.

373. The ***Unit*** is the *Degree,* which is $\frac{1}{360}$ part of the circumference of any circle.

FIG. 1.



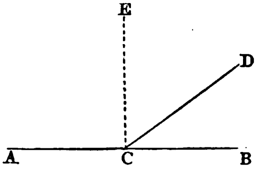
374. A ***Circle*** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the *Center.*

375. The ***Circumference*** of a circle is the line that bounds it.

376. An ***Arc*** is any part of the circumference; as AD, DE.

377. An ***Angle*** is the difference in the direction of two lines proceeding from a common point called the *vertex.* Thus, A C D and D C B are *angles,* and C is their vertex.

FIG. 2.



378. A ***Right Angle*** is formed by drawing one line perpendicular to another. Thus, A C E and E C B are right angles.

379. A ***Degree*** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided. Thus, E and B (Fig. 1) are at the distance of 90°, or a *right angle* from each other, the vertex being at the center of the circle.

380. The ***Measure of an Angle*** is the *arc* of the circle included between its sides. Thus, the arc D B (Fig. 3) is the measure of the angle D C B.

FIG. 3.

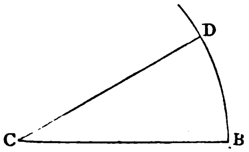


TABLE.

			Cir.	*S.*	°	′	″
60 Seconds (″)	= 1 Minute .	′	1=	12=	360=	21600=	1296000
60 Minutes	= 1 Degree .	°		1=	30=	1800=	108000
30 Degrees	= 1 Sign . .	*S.*			1=	60=	360
12 Signs, or 360°	= 1 Circle .	*Cir.*				1=	60

1. A ***Semi-Circum.*** is *one-half* of a circumference, or 180°.

2. A ***Quadrant*** is *one-fourth* of a circumference, or 90°.

3.. A ***Sextant*** is *one-sixth* of a circumference, or 60°.

4. A ***Sign*** is *one-twelfth* of a circumference, or 30°.

5. A *degree* varies with the size of the circle; thus, a degree of long. at the Equator is 69.16 statute miles, at 30° of latitude it is 59.81 mi., at 60° of latitude it is 34.53 mi., and at 90°, it is nothing.

6. A *minute* of the earth's circumference is called a geographic, *or nautical mile,* and is a small fraction less than 1.16 common miles.

COUNTING.

381. The following table is used in counting certain classes of articles:

12 Units or things	= 1	Dozen . . .	*doz.*
12 Dozen	= 1	Gross . . .	*gro.*
12 Gross	= 1	Great Gross .	*G. gro.*
20 Units or things	= 1	Score . . .	*Sc.*

1. *Two* things of a kind are often called a ***pair***, and ***six*** things a ***set***; as a ***pair*** of horses, a ***set*** of chairs, spoons, etc.

PAPER.

382. The denominations of the following table are used in the paper trade:

24 Sheets	= 1	Quire.	1 Bale	=	5 Bundles.
20 Quires	= 1	Ream.	1 Bundle	=	2 Reams.
2 Reams	= 1	Bundle.	1 Ream	=	20 Quires.
5 Bundles	= 1	Bale.	1 Quire	=	24 Sheets.

BOOKS.

383. The terms *folio, quarto, octavo,* etc., indicate the number of leaves into which a sheet of paper is folded.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 leaves	a Folio,	4 pp. (pages).
4 "	a Quarto or 4to,	8 "
8 "	an Octavo or 8vo,	16 "
12 "	a Duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "

Clerks and copyists are usually paid by the *folio* for making copies of legal papers, records, and documents.

72 words make 1 folio, or sheet of common law.
90 " 1 " chancery.

ORAL EXERCISES.

384. 1. How many seconds in ½ min.? In ⅔ min.?

2. How many minutes in 120 sec.? In 180 sec.?

3. How many hours in 90 min.? In 200 min.?

4. How many hours in 2 da.? In 3⅔ da.? In 5½ da.?

5. How many hours from 6 A. M. to 5 P. M.?

6. In 4 wk. 3 da., how many days? In 5 wk. 4 da.?

7. How many minutes from 10 min. past 9 o'clock to 25 min. past 10 A. M.?

8. How much time from 20 minutes before 11 A. M. to half past 10 o'clock P. M.?

9. Which of the months have 30 da. each? 31 da. each?

10. How many days from Jan. 1 to March 5, inclusive?

11. How many days from May 10 to July 16, inclusive?

12. Which of the following are leap years, and which are common years: 1874? 1876? 1880? 1886? 1900?

13. How many centuries and years since the birth of Christ?

14. How many leap years in every century?

15. How many degrees in ½ a circle? In ⅓? In ¼? In ⅙?

16. How many geographic miles in 2°? In 3°? In 4°?

17. How many common miles in 6 geographic miles?

18. How many degrees in 360 nautical miles?

19. How many degrees in ½ a quadrant? In ½ a sextant?

20. How many degrees in ¼ of a circumference?

21. What part of a circumference are 60°? 90°? 180°?

22. How many dozens in 2½ gross? In 3¾ gro.?

23. How many dozens in ½ of a great gross? In ¾?

24. How many score in 100? Pairs in 50? Sets in 75?

25. In 1 B. of paper, how many reams? How many *quires?*

26. How many eggs in 5¾ dozen? In 12 doz. and 7?

27. How many quires of paper in ½ a ream? In 3¼ rm.?

28. How many years in 4 score years and 10?

29. How many sheets of paper will be required to make a 12mo book of 320 pages? Of 480 pages?

30. How many sheets will be required to make a quarto book of 144 pages? Of 240 pp.? Of 360 pp.?

31. How many 16mo books will the paper for 1 quarto book make?

MEASURES OF VALUE.

385. ***Money*** is the measure of the *value* of things, and is used as a medium of exchange in trade?

386. ***Specie,*** or ***Coin,*** is metal struck, stamped, or pressed with a die, to give it a fixed legal value, and authorized by Government to be used as money.

387. ***Paper Money*** consists of bills and notes duly authorized by Government to circulate as substitutes, or representatives of money.

388. ***Currency*** is a term applied to all kinds of money employed in trade and commerce, both of coin and paper.

389. A ***Mint*** is a place in which the coin of a country or government is manufactured.

390. An ***Alloy*** is a compound of any metal with another of less value. In coinage, the less valuable or *baser metal* is not reckoned of any value.

Gold and silver, in a pure state, are too soft for coinage; hence they are hardened by compounding them with an alloy of baser metal, while their color and other valuable qualities are not impaired.

UNITED STATES MONEY.

391. ***United States Money*** is the legal currency of the United States, and is sometimes called *Federal Money*.

392. The ***Unit*** of U. S. Money is the *Gold Dollar*.

TABLE.

10 Mills (*m.*) = 1 Cent . . *ct.*
10 Cents = 1 Dime . . *d.*
10 Dimes = 1 Dollar . . $.
10 Dollars = 1 Eagle . . *E.*

E.	$	*d.*	*ct.*	*m.*
1 =	10 =	100 =	1000 =	10000
	1 =	10 =	100 =	1000
		1 =	10 =	100
			1 =	10

Federal money was adopted by Congress in 1786. Previous to this, pounds, shillings, and pence were in use.

393. The ***Coin*** of the United States consists of *gold, silver, nickel,* and *bronze,* and as fixed by the "New Coinage Act" of 1873, is as follows:

394. ***Gold.*** The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

395. ***Silver.*** The *Trade*-dollar, half-dollar, quarter-dollar, and the ten-cent piece.

396. ***Nickel.*** The five-cent, and three-cent pieces.

397. ***Bronze.*** The one-cent piece.

1. The *old* silver dollar, half-dime, and three-cent pieces, the bronze two-cent, and the nickel one-cent pieces are no longer coined.
2. The *Trade-dollar* is designed solely for purposes of commerce and not for currency. The weight is 420 grains.
3. The *Standard purity* of the *gold* and *silver* coins is, 9 parts (.9) pure metal, and 1 part (.1) alloy. The alloy for *gold* coins is *silver* and *copper,* the silver, by law, not to exceed $\frac{1}{10}$ of the whole alloy. The alloy of *silver* coins is *pure copper.*
4. The *five* and *three* cent pieces consist of 3 parts (.75) copper, and 1 part (.25) nickel.
5. The *one* cent piece consists of .95 copper, and .5 of zinc and tin.

CANADA MONEY.

398. ***Canada Money*** is the legal currency of the *Dominion* of Canada.

The denominations are *dollars, cents,* and *mills,* and have the same *nominal* value as the corresponding denominations of U. S. Money.

399. The ***Coin*** of the Dominion of Canada is *silver* and *bronze.*

400. The ***Silver Coins*** are the fifty-cent, twenty-five-cent, ten-cent, and five-cent pieces.

401. The ***Bronze Coin*** is the one-cent piece.

1. The *gold coins* in use are the *Sovereign* and the *Half-Sovereign.*

2. The intrinsic value of the 50-cent piece in United States coin is about $46\frac{1}{5}$ cents, of the 25-cent piece $23\frac{1}{10}$ cents. In ordinary business transactions, they pass the same as United States coin.

ENGLISH MONEY.

402. ***English,*** or ***Sterling, Money*** is the legal currency of Great Britain.

403. The ***Unit*** of Eng. Money is the *Pound Sterling.*

TABLE.

4 Farthings (*far.*) = 1 Penny . . . . *d.*
12 Pence = 1 Shilling . . . . *s.*
20 Shillings = { 1 Sovereign, or . . *sov.*
1 Pound . . . . £.

£.	*s.*	*d.*	*far.*
1=	20=	240=	960
	1=	12=	48
		1=	4

The value of a Sovereign in United States money is $4.8665.

OTHER DENOMINATIONS.

2 Shillings (*s.*) = 1 Florin . . . . *fl.*
5 Shillings = 1 Crown . . . . *cr.*

404. The ***Coin*** of Great Britain in general use consists of *gold, silver,* and *copper,* as follows :

405. ***Gold.*** The sovereign and half-sovereign.

406. ***Silver.*** The crown, half-crown, florin, shilling, six-penny, and three-penny piece.

407. ***Copper.*** The penny, half-penny, and farthing.

FRENCH MONEY.

408. ***French Money*** is the legal money of France and is á *decimal* currency.

409. The ***Unit*** of French Money is the *Silver Franc.*

TABLE.

10 Millimes (*m.*) = 1 Centime . . *ct.*
10 Centimes = 1 Decime . . *dc.*
10 Decimes = 1 Franc . . . *fr.*

fr.	*dc.*	*ct.*	*m.*
1 =	10 =	100 =	1000
	1 =	10 =	100
		1 =	10

The value of a franc in U. S. money of account is $.193.

410. The ***Coin*** of France consists of *gold, silver,* and *bronze.*

411. ***Gold.*** The 100, 40, 20, 10, and 5 franc pieces.

412. ***Silver.*** The 5, 2, and 1 franc, the 50 and the 25 centime pieces.

413. ***Bronze.*** The 10, 5, 2, and 1 centime pieces.

GERMAN MONEY.

414. The ***New Empire of Germany*** has adopted a new and uniform system of coinage.

415. The ***Unit*** is the "*Mark*" (*Reichsmark*), equal to 23.85 cents, U. S. Money.

A pound of gold .900 fine is divided into 139½ pieces, and the $\frac{1}{10}$ part of this gold coin is called a "Mark," and this is subdivided into 100 pennies (*Pfennige*).

416. The ***Coin*** of the New Empire consists of *gold, silver,* and *nickel,* and is as follows :

417. ***Gold.*** The 20, the 10, and the 5-mark pieces.

418. ***Silver.*** The 2, and the 1-mark, and the 20-penny pieces.

419. ***Nickel.*** The 10, and the 5-penny, and pieces of less valuation.

The 10-mark piece (gold) is equal to 3⅓ P. Thalers (old).

The 1-mark (silver) is equal to 10 S. Groschen, or 100 pennies.

The 20-penny (silver) is equal to 2 S. Groschen, or ⅕ of a mark.

The 10-penny (nickel) is equal to 1 S. Groschen, or $\frac{1}{10}$ of a mark.

420. SYNOPSIS FOR REVIEW.

- DENOMINATE NUMBERS—CONTINUED.
  - MEASURES—*Continued.*
    - 6. MEASURES OF TIME.
      - 1. DEFINITION OF TIME.
      - 2. STANDARD UNIT. 3. TABLE.
      - 4. RULE FOR LEAP YEAR, I, II.
    - 7. CIRCULAR MEASURE.
      - 1. FOR WHAT USED.
      - 2. STANDARD UNIT.
      - 3. DEFS.
        - 1. Circle. 2. Circumference. 3. Arc. 4. Right Angle. 5. Degree.
      - 4. MEASURE OF AN ANGLE. 5. TABLE.
    - 8. MISCELLANEOUS MEASURES.
      - 1. COUNTING. Table.
      - 2. PAPER. Table.
      - 3. BOOKS. Table.
    - 9. MEASURES OF VALUE.
      - 1. DEFS.
        - 1. Money. 2. Specie. 3. Paper Money. 4. Currency. 5. Mint. 6. Alloy.
      - 2. U. S. MONEY.
        - 1. Definition.
        - 2. Unit. 3. Table.
        - 4 Coin.
      - 3. CANADA MONEY.
        - 1. Definition.
        - 2. Denominations.
        - 3. Coin.
      - 4. ENGLISH MONEY.
        - 1. Definition.
        - 2. Unit. 3. Table.
        - 4. Coin.
      - 5. FRENCH MONEY.
        - 1. Definition.
        - 2. Unit. 3. Table.
        - 4. Coin.
      - 6. GERMAN MONEY.
        - 1. Definition.
        - 2. Unit.
        - 3. Coin.

REDUCTION.

421. ***Reduction of Denominate Numbers*** is the process of changing their denomination without altering their value.

422. Denominate numbers may be changed from higher to lower denominations, or from lower to higher denominations.

423. To reduce denominate numbers from higher to lower denominations.

ORAL EXERCISES.

1. How many inches in 3 ft.? In 5 ft.? In 4 ft. 10 in.?
2. How many feet in 5 yd. 2 ft.? In 1 rd. 10 ft.?
3. Reduce 12 fath. 4 ft. to feet. 15½ hands to inches.
4. How many quarters in 3½ yd.? How many eighths?
5. How many chains in 1½ mi.? How many rods?
6. In 3½ sq. yd., how many sq. ft.? In 7 sq. yd. 5 sq. ft.?
7. In 10 A., how many sq. ch.? How many sq. rd.?
8. Change 3 cu. yd. to cu. ft. 3 cd. ft. to cubic feet.
9. Change 4 cords to cord feet. 2 perch to cubic feet.
10. How many quarts in 2 gal. 3 qt.? In 5½ gal.?
11. In 4 bu. 1 pk., how many pecks? Quarts? Pints?
12. In 2 pints, how many fluidounces? Fluidrachms?
13. How many pints in 3 pk.? In 2 pk. 6 qt.?
14. How many half-pecks in 1½ bu.? In 3¼ bu.?
15. In 5 lb. Troy, how many ounces? In 5 lb. Avoir.?
16. How many pounds in 5 cwt. 20 lb.? In 4⅗ cwt.?
17. In 5 dr., how many scruples? How many grains?
18. In 2 bu. 20 lb. of wheat, how many pounds?

19. How many minutes in 5 hr. 40 min.? In $4\frac{1}{2}$ hr.?
20. In $\frac{1}{2}$ a sign, how many degrees? Geographic miles?
21. In 10 gross 9 doz., how many dozen? In $7\frac{1}{2}$ gro.?
22. In 2 reams of paper how many quires? Sheets?

23. In £5 10s., how many shillings? In $3\frac{1}{2}$ sov.?
24. In 5 francs how many centimes? In $10\frac{1}{2}$ francs?
25. How many pence in $1\frac{1}{2}$ crowns? In 12 florins?
26. How many crowns in £5? How many florins?
27. How many pennies in 3 marks? In $5\frac{1}{2}$ marks?

424. PRINCIPLE.—*Denominate Numbers are changed to lower denominations by* MULTIPLICATION.

WRITTEN EXERCISES.

425. 1. Reduce 33 rd. 4 yd. 2 ft. 10 in. to inches.

OPERATION.

$$\begin{array}{l} 28 \text{ rd. } 4 \text{ yd. } 2 \text{ ft. } 10 \text{ in.} \\ \underline{\quad 5\frac{1}{2}} \\ 158 \text{ yd.} \\ \underline{\quad 3} \\ 476 \text{ ft.} \\ \underline{\quad 12} \\ 5722 \text{ in.} \end{array}$$

ANALYSIS.—Since 1 rd. is equal to $5\frac{1}{2}$ yd., there are $5\frac{1}{2}$ times as many yards as rods, to which add the given number of yards; $\overline{28 \times 5\frac{1}{2}} + 4 = 158$, the number of *yards* in 28 rd. 4 yd.

Since 1 yd. is equal to 3 ft., there are 3 times as many feet as yards, to which add the given number of feet; $\overline{158 \times 3} + 2 = 476$, the number of *feet* in 28 rd. 4 yd. 2 ft.

Since 1 ft. is equal to 12 in., there are 12 times as many inches as feet; $\overline{476 \times 12} + 10 = 5722$, the number of *inches* in 28 rd. 4 yd. 2 ft. 10 inches.

2. Reduce 7 lb. 10 oz. 16 pwt. 11 gr. to grains.
3. In 3 T. 6 cwt. 21 lb. 12 oz., how many ounces?.
4. How many inches in 12 fathoms 3 ft. 10 in.?
5. Change 6 wk. 5 da. 9 hr. 25 min. to minutes.

RULE.—I. *Multiply the units of the highest denomination of the given number by that number of the scale that will reduce it to the next lower denomination, and to the product add the number of that denomination given.*

II. *Proceed in like manner with this and each successive denomination obtained, until the number is reduced to the required denomination.*

Reduce

6. 12 mi. 36 rd. 10 ft. to ft.
7. 10 rd. 5½ ft. to inches.
8. 27¾ yd. to eighths.
9. 1 A. 15 sq. yd. to sq. ft.
10. 2 sq. mi. 125 A. to P.
11. 14 sq. mi. to acres.
12. 3 mi. 51 ch. 6 l. to links.
13. 75 Cd. 6 cd. ft. to cu. ft.
14. 12 hhd. 21 gal. to pt.
15. 24 bu. 3 pk. to quarts.
16. Cong. 4, O. 5, f℥ 8 to fʒ.
17. 31½ gal. to gills.

Change

18. 7 T. 9 cwt. 18 lb. to lb.
19. 22 lb. 10 oz. to pwt.
20. ℔ 16, ℥ 7, ʒ 3, to ℈.
21. 1 common year to min.
22. The summer mos. to sec.
23. 1 leap year to hours.
24. 10 S. 22° 5′ to min.
25. 5 bundles to quires.
26. 6 Gr. gross to dozens.
27. 326½ sov. to pence.
28. 26½ fr. to centimes.
29. £34½ to pence.

30. How much is 5 lb. 9 oz. 14 pwt. of gold dust worth, at $.75 a pwt.?

31. How many rods of fence will enclose a farm ¾ of a mile square?

32. If 1 barrel will hold 2 bu. 3 pk., how many barrels will be required to hold 1548 bu. 1 pk.?

33. How many boxes, each containing 12 lb., can be filled from a hogshead containing 9 cwt. 60 lb. of sugar?

34. If I buy 9 bu. of chestnuts at $4¾ a bushel, and retail them at 12½ cents a pint, what is my whole gain?

35. How many times will a wheel 16½ ft. in circumference revolve in running 42 miles?

36. How many minutes less in every autumn of a common year than in either spring or summer?

37. If it require 4 reams 10 quires of paper to print a book, how many sheets are required?

38. At 12½ cents each, what will be the cost of 2 great gross of writing books?

39. If a clock tick seconds, how many times will it tick during February, 1877?

40. If your age is 21 yr. 26 da., how many minutes old are you, if 5 leap years have occurred in that time?

41. If a vessel sail 120 leagues in a day, how many statute miles does she sail?

42. How many pint, quart, and 2-quart bottles, of each an equal number, can be filled from a barrel of 31½ gallons?

43. In the eighteenth century, how many hours?

44. How large an edition of a 12mo book can be printed from 2 bales, 2 bundles, 15 quires of paper, allowing 8 sheets to the volume?

45. How many pages, 2 pages to each leaf, will there be in an 8vo book, containing 16 fully printed sheets?

How many pounds

46. In 36¼ centals of grain?
47. In 42$\frac{4}{7}$ bbl. of flour?
48. In 29.5 quintals of fish?
49. In 116½ bbl. of salt?
50. In 63.25 kegs of nails?
51. In .75 of 75 bu. of salt?
52. In 125¾ bu. of wheat?
53. In ⅔ of 21648 bu. oats?
54. In .7 of 40 bu. corn meal?
55. In 7.5 casks of lime?

What is the value in U. S. Money

56. Of 28 sovereigns?
57. Of £25 10s.?
58. Of 25 francs?
59. Of 42½ marks?

426. To reduce denominate numbers from lower to higher denominations.

ORAL EXERCISES.

1. How many feet in 108 in.? How many yards?
2. How many square yards in 63 sq. ft.? In 85 sq. ft.?
3. How many chains in 200 l.? In 425 l.? In 674 l.?
4. In 81 cu. ft., how many cu. yd.? How many cd. ft.?
5. How many cords in 100 cd. ft.? In 256 cu. feet?
6. Change 120 sq. ch. to A. 80 P. to square chains.
7. In 162 in., how many hands? Spans? Feet?

8. In 112 pt., how many quarts? Pecks? Bushels?
9. How many gallons in 46 qt.? 96 pt.? 128 gi.?
10. Reduce O. 160 to Cong.; f ʒ 90 to f ℥.
11. Change 96 oz. to Troy pounds; to Avoir.
12. Reduce ℈ 45 to oz.; ℥ 75 to pounds.
13. In 400 pwt., how many oz.? How many lb.?
14. In 508 lb., how many cwt.? In 1276 lb.?
15. In 630 lb. of wheat, how many bushels?

16. In 140 da. how many wk.? Months, of 30 da. each?
17. Change 1200 min. to hours. 84 hr. to days.
18. How many doz. are 240 eggs? How many gross?
19. How many degrees in 180′? Minutes, are 240″?
20. In 90 objects, how many score? Sets? Pairs?
21. In 120 quires of paper, how many reams? Bundles?
22. In 120d. how many shillings? Crowns? Florins?
23. In 500 pennies, how many marks?

427. PRINCIPLE.—*Denominate numbers are changed to higher denominations by* DIVISION.

WRITTEN EXERCISES.

428. 1. Change 5722 inches to rods.

OPERATION.

12)5722 in.
3)476 ft. + 10 in.
5½)158 yd. + 2 ft.
2   2
11)316 half-yd.
28 rd. + 4 yd.
5722 in. = 28 rd. 4 yd. 2 ft. 10 in.

ANALYSIS.—Since 12 in. make 1 ft., in 5722 in. there are as many feet as 12 in. are contained times in 5722 in., or 476 ft. and 10 in. more.

Since 3 ft. make 1 yd., in 476 ft. there are 158 yd. and 2 ft. more.

And since 5½ yd. make 1 rd., in 158 yd. there are 28 rd. and 4 yd. more.

In order to divide by 5½, both dividend and divisor may be reduced to *halves* before dividing. In this case the remainder, if any, is *halves*, which may be reduced to *integers*.

2. Reduce 157540 minutes to weeks.
3. Reduce 80820 links to miles.
4. Change 487630 pwt. to pounds.

RULE.—I. *Divide the units of the given denomination by that number of the scale which is equal to a unit of the next higher denomination, and write the remainder as a part of the answer.*

II. *In like manner, divide this and each successive quotient until reduced to the denomination required. The last quotient, with the remainders annexed, is the required result.*

How many
5. Miles are 3168000 in.?
6. Acres are 256800 P.?
7. Sq. mi. are 27878400 sq. ft.?
8. Cu. ft. are 216840 cu. in.?
9. Cords are 38042 cu. ft.?

Reduce
10. 30876 gills to hhd.
11. 27072 qt. to bushels.
12. 66742 pt. to barrels.
13. 103720 pt. to gallons.
14. ℥ 8106 to Cong.

How many

15. Pounds Troy are 85894 gr.?
16. Tons are 51570 pounds?
17. Cwt. are 40607 ounces?
18. Pounds are 3000 pwt?
19. Bu. are 12060 lb. of wheat?
20. Bbl. are 3038 lb. of flour?
21. Bu. are 6496 lb. of oats?
22. Quin. are 3172 lb. of fish?

23. Weeks are 3114061 sec.?
24. Months are 8263420 min.?
25. Degrees are 2007200″?
26. Deg. are 5270 Naut. mi.?

Reduce

27. 120400 pens to gro.
28. 2734 eggs to dozens.
29. 5020 balls to scores.
30. 10738 sheets to rm.
31. 6048 quires to bun.

32. 24684d. to crowns.
33. 4076s. to florins.
34. $194.66 to half-sov.
35. 42346 far. to £.
36. $86.85 to francs.
37. $225.40 to sov.
38. $57.70 to marks.

39. If the Atlantic Cable is 3200 mi. in length, and cost 10 cents a foot, what was its entire cost?

40. If a cubic foot of gray limestone weigh 175 lb., what is the weight of a cubic yard?

41. What is the cost of a load of oats weighing 1860 lb., at $.56 a bushel?

42. In a storm at sea, a ship changed her longitude 423 geographic mi. How many degrees and minutes?

43. How much time will a person gain in 40 yr., by rising 25 min. earlier and retiring 20 min. later every day, counting 9 leap years in the time?

44. What will a peck of clover-seed cost, at $.12½ a lb.?

45. What will a ton of corn-meal cost, at $1.20 a bu.?

46. An Illinois farmer sold a load of corn weighing 2496 lb., and a load of oats weighing 1920 lb.; for the corn he received $.62 a bushel, and for the oats $.44 a bushel. What did he receive for both loads?

REDUCTION OF DENOMINATE FRACTIONS.

429. A ***Denominate Fraction*** is a fraction whose integral *unit* is a denominate number. Thus, $\frac{6}{7}$ of a week, .7 of an acre, are denominate fractions.

The Principles, Operations, and Analyses of the reduction of *denominate fractions* are essentially the same as those of *denominate integers.*

430. To reduce denominate fractions from higher to fractions of lower denominations.

ORAL EXERCISES.

1. Reduce $\frac{1}{16}$ of a gallon to the fraction of a pint.

ANALYSIS.—Since in 1 gal. there are 4 qt., in $\frac{1}{16}$ gal. there are $\frac{1}{16}$ of 4 qt., or $\frac{1}{4}$ qt.; and since in 1 qt. there are 2 pt., in $\frac{1}{4}$ qt. there are $\frac{1}{4}$ of 2 pt., or $\frac{1}{2}$ pt. Hence $\frac{1}{16}$ gal. equals $\frac{1}{2}$ pt.

2. Reduce $\frac{1}{24}$ lb. Troy to the fraction of an oz.
3. What part of a pint is $\frac{3}{10}$ of a qt.? $\frac{1}{18}$ of a pk.?
4. What decimal part of a day is .12 of a week?

ANALYSIS.—Since in 1 wk. there are 7 days, in .12 wk. there are .12 of 7 da., or .84 da. Hence .12 wk. equals .84 da.

5. What part of a peck is .02 of a bu.? .07 bu.? .25 bu.?
6. Reduce .5 gal. to the fraction of a quart? Of a pint?
7. What part of an inch is $\frac{1}{15}$ of a foot? $\frac{1}{40}$ of a yard?
8. Change .04 of a pound to the decimal of an ounce.

WRITTEN EXERCISES.

431. 1. Reduce $\frac{1}{112}$ of a bushel to the fraction of a pint.

OPERATION.

$\frac{1}{112}$ bu. $\times 4 = \frac{1}{28}$ pk.

$\frac{1}{28}$ pk. $\times 8 = \frac{2}{7}$ qt.

$\frac{2}{7}$ qt. $\times 2 = \frac{4}{7}$ pt.

Or, $\frac{1}{\cancel{112}} \times \frac{4}{1} \times \frac{\cancel{8}}{1} \times \frac{\cancel{2}}{1} = \frac{4}{7}$ pt.

ANALYSIS.—Same as for oral questions. (**430.**)

Multiply successively by 4, 8, and 2, the numbers in the descending scale required to reduce bushels to pints. (**425.**)

2. Reduce $\frac{1}{132}$ of a rod to the fraction of a foot.
3. Change $\frac{1}{960}$ of an ounce to the fraction of a grain.
4. What part of a pint is $\frac{1}{672}$ of a hogshead?
5. What part of a shilling is .012 of a £?

RULE.—*Multiply the fraction of the higher denomination by the numbers as factors in the descending scale successively between the given and the required denomination.* **(425.)**

6. What part of an ounce is $\frac{3}{1280}$ of a pound Avoir.?
7. Reduce $\frac{7}{1440}$ of an acre to the fraction of a sq. rd.
8. Reduce .005 of a bushel to the decimal of a pint.
9. What part of a yard is $\frac{6}{7}$ of $\frac{4}{11}$ of a rod?
10. Change .0000625 mi. to the decimal of a foot.
11. What part of an ounce Troy is $\frac{2}{9}$ of $\frac{1}{6}$ of 2 pounds?
12. What part of a yard is $\frac{1}{2112}$ of a mile?
13. What fraction of a link is $\frac{1}{45}$ of a rod?
14. What part of a minute is .000175 of a day?
15. What part of a sq. rd. is $\frac{3}{1360}$ of $4\frac{1}{2}$ times $\frac{2}{15}$ A.?

432. To reduce denominate fractions to integers of lower denominations.

ORAL EXERCISES.

1. How many hours in $\frac{2}{3}$ of a day?

ANALYSIS.—Since in 1 da. there are 24 hr., in $\frac{2}{3}$ of a day there are $\frac{2}{3}$ of 24 hr., or 16 hr. Hence $\frac{2}{3}$ da. equals 16 hr.

2. How many minutes in $\frac{5}{12}$ hr.? In $\frac{4}{15}$ hr.? In $\frac{3}{4}$ hr.?
3. How many quarts in $\frac{5}{8}$ pk.? In $\frac{1}{4}$ bu.? In $\frac{7}{16}$ bu.?
4. How many ounces in .5 of a pound?

ANALYSIS.—Since in 1 lb. there are 16 oz., in .5 lb. there are .5 of 16 oz., or 8 oz. Hence .5 lb. equals 8 oz.

5. How much is .7 hr.? .25 hr.? .15 hr.? .8 hr.?
6. How many yards in $\frac{3}{11}$ of a rod? In $\frac{2}{3}$ of a rod?
7. How many cwt. in $\frac{2}{5}$ of a ton? How many pounds?
8. Change to pints $\frac{1}{2}$ gal. $\frac{1}{4}$ pk. $\frac{3}{16}$ bu. $\frac{3}{4}$ of 2 pk.
9. Change $\frac{1}{4}$ of an acre to sq. rd. $\frac{2}{7}$ sq. yd. to sq. ft.
10. How many pecks in .75 bu.? Quarts in 1.25 pk.?
11. Change $\frac{3}{4}$ lb. to oz. .45 oz. to pwt. .53 cwt. to lb.

WRITTEN EXERCISES.

433. 1. Reduce $\frac{5}{6}$ bu. and .645 da., each to integers of lower denominations.

1ST OPERATION.	1ST OPERATION.
$\frac{5}{6}$ bu. × 4 = $\frac{20}{6}$ = $3\frac{1}{3}$ pk.	.645 da. × 24 = 15.48 hr.
$\frac{1}{3}$ pk. × 8 = $\frac{8}{3}$ = $2\frac{2}{3}$ qt.	.48 hr. × 60 = 28.8 min.
$\frac{2}{3}$ qt. × 2 = $\frac{4}{3}$ = $1\frac{1}{3}$ pt.	.8 min. × 60 = 48 sec.
$\frac{5}{6}$ bu. = 3 pk. 2 qt. $1\frac{1}{3}$ pt.	.645 da. = 15 hr. 28 min. 48 sec.

1. The analyses of the above are the same as in (**425**) and (**430**).

2. The following methods may be regarded as most convenient in practice, since the operations are performed without *rewriting the fractional part of each product.*

2D OPERATION.

```
           5
           4
        ----
        6)20(3 pk. 2 qt. 1⅓ pt.
  4       18
          --
  2        2
 --
6)8 pt.    8    ⅚ bu.=3 pk.2 qt.1⅓ pt.
  6     ----
 --     6)16 qt.
  2       12
          --
           4
```

2D OPERATION.

```
 .645 da.
   24
 ----
 2580
1290
 ------
15.480 hr.
    60
 ------
28.800 min.
    60
 ------
48.000 sec.
```

.645 da. = 15 hr. 28 min. 48 sec.

Reduce to integers of lower denominations,

2. $\frac{13}{24}$ of a £.
3. .35 lb. Apoth.
4. $\frac{4}{15}$ of a mi.
5. .75 lb. Troy.
6. .625 of a fath.
7. .55 lb. Avoir.

RULE.—I. *Multiply the given fraction or decimal by that number in the scale that will reduce it to the next lower denomination.* **(425.)**

II. *Proceed in like manner with the fractional part of each successive product, until it is reduced to the denomination required.*

III. *The integral parts of the several products, arranged in their proper order, is the required result.*

Find the value in *integers* of lower denominations,

8. Of $\frac{4}{7}$ mo.
9. Of .555 £.
10. Of $\frac{7}{13}$ A.
11. Of $\frac{2}{3}$ of $\frac{5}{8}$ lb.
12. Of $\frac{8}{15}$ cu. yd.
13. Of .1934 S.
14. Of f ℥ .7.
15. Of $\frac{607}{640}$ T.
16. Of .875 hhd.
17. Of $\frac{5}{9}$ sq. rd.
18. Of ℔ $\frac{15}{22}$.
19. Of $\frac{4}{7}$ Gr. gro.
20. Of .67 lea.
21. Of .125 bbl.
22. Of .578125 bu.
23. Of .6625 mi.
24. Of $\frac{7}{9}$ of $5\frac{1}{2}$ T.
25. Of $\frac{3}{8}$ of $3\frac{2}{3}$ A.
26. Of $\frac{4}{5}$ of $3\frac{3}{8}$ Cd.
27. Of $\frac{4}{5}$ of .225 mi.
28. Of .3125 ream.

29. At $8\frac{1}{3}$ cents a pound, what will $\frac{13}{40}$ T. of cheese cost?

434. To reduce denominate fractions from lower to fractions of higher denominations.

ORAL EXERCISES.

1. Reduce $\frac{2}{3}$ of a peck to the fraction of a bushel.

ANALYSIS.—Since 4 pk. make 1 bu., there are $\frac{1}{4}$ as many bushels as pecks; $\frac{1}{4}$ of $\frac{2}{3}$ pk. $= \frac{2}{12}$ or $\frac{1}{6}$ bu. Hence $\frac{2}{3}$ pk. equals $\frac{1}{6}$ bu.

2. Reduce $\frac{4}{5}$ of a pint to the fraction of a gallon.
3. What part of a pound Avoir. is $\frac{2}{5}$ oz.?
4. What part of a week is $\frac{3}{5}$ da.? $\frac{7}{9}$ da.? $\frac{2}{3}$ da.? $\frac{5}{6}$ da.?

5. What decimal part of a gallon is .28 of a quart?

ANALYSIS.—Since 4 qt. make 1 gal., there are $\frac{1}{4}$ as many gallons as quarts; $\frac{1}{4}$ of .28 qt. is .7 gal. Hence .28 qt. equals .7 gal.

6. Change .32 of a pint to the decimal of a quart.

7. What decimal of a pound Troy is .48 oz.? .84 oz.?

8. What decimal of a week is .35 da.? .63 da.?

9. Change .72 in. to the decimal of a foot. Of a yd.

WRITTEN EXERCISES.

435. 1. What fraction of a bushel is $\frac{4}{5}$ of a pint?

OPERATION.

$\frac{4}{5}$ pt. $\div 2 = \frac{2}{5}$ qt.

$\frac{2}{5}$ qt. $\div 8 = \frac{1}{20}$ pk.

$\frac{1}{20}$ pk. $\div 4 = \frac{1}{80}$ bu.

Or, $\frac{4}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{80}$ bu.

ANALYSIS. — Divide successively by 2, 8, and 4, the numbers in the ascending scale required to reduce pints to bushels. (**428.**) Hence $\frac{4}{5}$ pt. $= \frac{1}{80}$ bu.

2. Reduce $\frac{8}{11}$ of a gill to the fraction of a gallon.

3. Change $\frac{2}{9}$ of a shilling to the fraction of a £.

4. Reduce .64 of a pint to the decimal of a bushel.

5. What part of a pound Troy is .576 of a grain?

RULE.—*Divide the fraction of the lower denomination by the numbers as factors in the ascending scale successively between the given and the required denomination.* (**428.**)

6. What decimal of a ton is .8 lb.? .36 of a cwt.?

7. Reduce $\frac{12}{49}$ of a cord foot to the fraction of a cord.

8. Reduce .216 gr. to the decimal of an ounce Troy.

9. What part of a ton is $\frac{5}{7}$ of a pound?

10. What part of a day is $\frac{9}{4}$ of a minute? .12 hr.?

11. What decimal of a rod are 3.96 in.?

12. How much less is $\frac{3}{5}$ of a pint than $\frac{1}{630}$ of a hhd.?

13. What part of an acre is $\frac{7}{9}$ of a square rod?

436. To reduce a compound denominate number to a fraction of a higher denomination.

ORAL EXERCISES.

1. What part of a pound are 4 oz.? 8 oz.? 10 oz.?
2. What part of a foot are 9 in.? What part of a yard?
3. What part of a bushel are 2 pk. 4 qt.?

ANALYSIS.—1 bu. = 32 qt., and 2 pk. 4 qt. = 20 qt.; 20 qt. = $\frac{20}{32}$ bu. = $\frac{5}{8}$ bu., or .625 bu. Hence 2 pk. 4 qt. = $\frac{5}{8}$ bu., or .625 bu.

4. What part of a yard are 2 ft. 6 in.? Are 18 in.?
5. What part of 3 lb. Troy are 1 lb. 6 oz.? Are 9 oz.?
6. What part of 5 gal. are 2 gal. 2 qt.? 3 gal. 1 qt.?
7. Reduce 12 oz. to the decimal of 3 lb. Avoir.
8. What fraction of 3 Cd. 6 cd. ft. are 2 Cd. 4 cd. ft.?
9. What part of 3 pk. are 1 pk. 4 qt.? Are $2\frac{1}{2}$ pk.?

WRITTEN EXERCISES.

437. 1. What decimal of a pound Troy are 2 oz. 14 pwt.?

1ST OPERATION.

20)14 pwt.
12)2.7 oz.
.225 lb. = $\frac{9}{40}$ lb.

ANALYSIS.—Since 20 pwt. make 1 oz., there are $\frac{1}{20}$ as many ounces as pennyweights; and $\frac{1}{12}$ as many pounds as ounces (**428**). Hence 2 oz. 14 pwt. = .225 lb., or changed to a *fraction*, by **283**, is $\frac{9}{40}$ lb.

2D OPERATION.

2 oz. 14 pwt. = 54 pwt.
1 lb. = 240 pwt.
$\frac{54}{240} = \frac{9}{40}$ lb. = .225 lb.

ANALYSIS.—In order to find *what part* one compound number is of another, *both* must be *like numbers*, and must be reduced to the lowest denomination in *either*. Thus, 2 oz. 14 pwt. are equal to 54 pwt., and 1 lb. is equal to 240 pwt. Hence 2 oz. 14 pwt. = $\frac{54}{240}$ lb. = $\frac{9}{40}$ lb., or, reduced to a *decimal*, by **285**, .225 lb.

2. Reduce 3 gal. 3 qt. $1\frac{1}{2}$ pt. to the fraction of a bbl.
3. Reduce 3 cd. ft. 8 cu. ft. to the decimal of a cord.

RULE.—I. *Divide the units of the lowest denomination given by that number in the scale which is equal to a unit of the next higher denomination, and annex the quotient as a decimal to the number given of that denomination.*

II. *Proceed in like manner until the whole is reduced to the denomination required.* Or,

Reduce the given number to its lowest denomination for the numerator of the required fraction, and a unit of the required denomination to the same denomination for the denominator, and reduce the fraction to its lowest terms, or to a decimal.

1. If the given number contain a fraction, the *denominator* of this fraction must be regarded as the *lowest* denomination.

2. The pupil may be required to give the answers either in the form of a fraction, or of a decimal, or both.

4. Reduce 13 gal. 3 qt. 3.62 gi. to the decimal of a hhd.

5. What part of a pound Troy are 10 oz. 13 pwt. 9 gr.?

6. What fraction of 2 T. 7 cwt. 28 lb. are 5 cwt. 91 lb.?

7. What part of 3 A. 80 P. are 51.52 P.?

8. What part of a f ℥ are f ʒ 5 ♏ 36?

9. Change 126 A. 4 sq. ch. 12 P. to the decimal of a Tp.

10. What decimal part of 25° 42′ 40″ are 7° 42′ 48″?

11. From a hhd. of molasses 28 gal. 2 qt. were drawn. What part of the whole remained?

12. What decimal of a league are 2 mi. 3 rd. 1 yd. $3\frac{3}{8}$ in.?

13. What part of 3 bbl. of flour are 110 lb. 4 oz.?

14. What decimal part of a ton is $\frac{1}{7}$ of $22\frac{3}{4}$ lb.?

15. Reduce .45 pk. to the decimal of $1\frac{1}{4}$ bu.

16. What part of 54 cords of wood are 4800 cu. ft.?

17. Change 18s. 5d. $2\frac{2}{13}$ far. to the fraction of a £.

REVIEW.

WRITTEN EXAMPLES.

438. 1. How many steps of 30 in. each must a person take in walking 21 miles?

2. How long will it take one of the heavenly bodies to move through a sextant, at the rate of 3′ 12″ a minute?

3. Reduce £10 18s. 6d. to United States Money.

4. Paid $425.75 for 2½ tons of cheese, and retailed it at 12½ cents a pound. What was the whole gain?

5. Reduce 580 francs to United States Money.

6. Change $291.99 to Sterling Money.

7. What cost 30 bu. 2 pk. 1 qt. of beans, at $4.20 a bu.?

8. Bought 15 cwt. 22 lb. of rice at $4.25 a cwt., and 6 cwt. 36 lb. of pearl-barley at $5.60 a cwt. What would be gained by selling the whole at 6¼ cents a pound?

9. How many bushels of corn in 36824 lb., Illinois standard? Louisiana? New York?

10. 5000 bu. of oats in Ohio are equal to how many bushels in Connecticut, by weight? In New Jersey?

11. If I buy 16 T. 3 cwt. 3 qr. 24 lb. of Eng. iron, by long ton weight, at 3d. a lb., and sell the same at $140, by the short ton, what do I gain by the transaction?

12. How many carats fine is a piece of gold ⅝ pure?

13. How many acres in a piece of land 105 ch. 85 l. long, and 40 ch. 15 l. wide?

14. If 10 lb. of milk make 1 lb. of cheese, what will it cost at 1 cent a pound to manufacture the cheese that can be made from 90000 lb. of milk?

15. At $75¾ an acre, what is the value of a farm 189.5 rd. long and 150 rd. wide?

16. What cost 2 bu. 3 pk. 6 qt. of green peas, at $.30 a peck?

17. What cost 3 T. 17 cwt. 20 lb. of hay, at $22¾ a ton?

18. If a grocer's scales give ½ oz. short of true weight on every pound, of how much money does he defraud his customers, in the sale of 3 bbl. of sugar, each weighing 2 cwt. 10 lb., at 12½ cents a pound?

19. If 37 A. 128 P. are sold from a farm containing 170 A. 16 P., what part of the whole remains?

20. Paid $526.05 for 3½ tons of cheese, and retailed it at 9½ cents a pound. How much was the whole gain?

21. How many bushels of oats in Connecticut are equivalent in weight to 2500 bushels in Iowa?

22. How many centals of barley in California are equivalent to 1500 bushels in Missouri?

23. A man sold 12 bu. 3 pk. 6 qt. of cranberries at $3½ a bushel, and took his pay in flour at 4 cents a pound. How many barrels should he receive?

24. If 3 T. 12 cwt. 20 lb. of ground plaster cost $15.75, what will be the cost of 5 T. 80 lb. at the same rate?

25. Bought 37 Cd. 48 cu. ft. of wood for $129.81, and there was but 13 Cd. 59 cu. ft. delivered. What part of the money should be paid?

26. If a grocer's gallon measure is too small by 1 gi., what does he make dishonestly in selling 2 hhd. of molasses, averaging 58 gal. 2 qt. 1 pt. each, worth $.80 a gal.?

27. How many reams of paper are required to supply 4500 subscribers with a weekly newspaper for 1 year?

28. A publisher printed an edition of 10000 copies of a 12mo book of 336 pp. How much paper did he use, allowing 1 quire to each ream for waste?

ADDITION.

439. Denominate numbers are *added, subtracted, multiplied,* and *divided* by the same general methods as are employed for like operations in Simple Numbers.

The corresponding processes are based upon the *same principles.* The only modification of the rules needed is that which is required by a *varying scale* instead of a *uniform scale* of 10.

The *principles* will be made sufficiently plain in the *operations* and *analyses* to render *special rules unnecessary.*

WRITTEN EXERCISES.

440. 1. Find the sum of 4 cwt. 46 lb. 12 oz., 12 cwt. $9\frac{1}{2}$ lb., $2\frac{1}{4}$ cwt., and $21\frac{5}{8}$ lb.

OPERATION.

cwt.	lb.	oz.
4	46	12
12	9	8
2	25	0
	21	10
19	2	14

ANALYSIS.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add.

The sum of the ounces is 30, equal to 1 lb. 14 oz. Write the 14 oz. under the column of ounces, and add the 1 lb. to the pounds of the next column.

The sum of the pounds is 102 lb., equal to 1 cwt. and 2 lb. Write the 2 lb. under the column of pounds, and add the 1 cwt. to the cwt. of the next column.

The sum of the cwt. is 19 cwt., which write under the column of cwt. Hence the entire sum is 19 cwt. 2 lb. 14 oz.

2. What is the sum of $\frac{7}{10}$ wk., $\frac{3}{5}$ da., and $\frac{3}{8}$ hr.?

1ST OPERATION.

	da.	hr.	min.	sec.
$\frac{7}{10}$ wk. =	4	21	36	00
$\frac{3}{5}$ da. =		14	24	00
$\frac{3}{8}$ hr. =			22	30
	5	12	22	30

ANALYSIS.—First find the value of each denominate fraction in integers of lower denominations (**433**), and then add the resulting compound numbers. Or, 2D OPERATION.

$\frac{3}{5}$ da. $= \frac{3}{35}$ wk.
$\frac{3}{8}$ hr. $= \frac{1}{448}$ wk.
$\frac{7}{10}$ wk. $+ \frac{3}{35} + \frac{1}{448} = \frac{353}{448}$ wk.
$\frac{353}{448}$ wk. = 5 da. 12 hr. 22 min. 30 sec.

Reduce the given fractions to fractions of the same denomination (434), then add the results and find the value of their sum in integers of lower denominations.

If denominate fractions occur in the given numbers, they should be reduced to integers of lower denominations (433) before adding.

3. Add 7 yd. 2 ft., 5 yd. 1¼ ft., 2 ft. 9½ in., 3 yd. 1 ft. 6½ in., 2¾ ft., and 4½ yd.

4. Add 5 Cd. 7 cd. ft., 2 Cd. 2 cd. ft. 12 cu. ft., 6 cd. ft. 15 cu. ft., 7⅜ Cd., and 3 Cd. 2 cu. ft.

5. What is the sum of 1⅔ hhd., 36 gal. 3 qt. 1¼ pt., ⅞ gal., 2 qt. ¾ pt., and 1.75 pt.?

6. What is the sum of ⅝ of a day added to $\frac{7}{15}$ of an hour?

7. To $\frac{4}{7}$ of a hhd. add ⅝ of 10 gal.

8. What is the sum of 22$\frac{4}{7}$ cwt., 26⅞ lb., and 14 oz.?

9. Add 5⅓ Pch., 18 cu. ft., 86.6 cu. ft., and ⅚ Pch.

10. Find the sum of ℔ 4 ℥ 6 ʒ 5, and ℔ 6 ℥ 4½ ʒ 9¾.

11. A Missouri farmer received $.75 a bushel for 4 loads of corn; the first contained 48.4 bu., the second 2626 lb., the third 36¾ bu., and the fourth 41 bu. 52 lb. What did he receive for the whole?

12. Bought three loads of hay at $15 a ton. The first weighed 1.125 T., the second 1⅜ T., and the third 2750 lb. What did the whole cost?

13. When B was born, A's age was 3 yr. 9 mo. 24 da.; when C was born, B's age was 12 yr. 19 da.; when D was born, C's age was 5 yr. 11 mo., and when E was born, D's age was 10 yr. 1 mo. 20 da. What was A's age on E's birthday?

SUBTRACTION.

WRITTEN EXERCISES.

441. 1. From 25 rd. 2 yd. 2 ft. 6.3 in., subtract 12 rd. 4 yd. 11.6 in.

OPERATION.

rd.	yd.	ft.	in.
25	2	2	6.3
12	4	0	11.6
12	3 (½)	1	6.7
	½ =	1	6
12	4	0	.7

ANALYSIS.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to subtract.

Since 11.6 in. cannot be subtracted from 6.3, take 1 ft., equal to 12 in., from the 2 ft., leaving 1 ft., and add it to the 6.3 in., making 18.6 in. Subtract 11.6 in., and write the remainder, 6.7 in., under the inches.

Since 1 ft. has been taken from 2 ft., subtract 0 ft. from 1 ft., and write the remainder 1 ft. under the feet.

Since 4 yd. cannot be taken from 2 yd., take 1 rd., equal to 5½ yd., from 25 rd., leaving 24 rd., and add it to the 2 yd., making 7½ yd. Subtract 4 yd. from 7½ yd., and write the remainder, 3½ yd., under the yards.

Since 1 rod has been taken from 25 rd., subtract 12 rd. from 24 rd., and write the remainder, 12 rd., under the rods.

The ½ yd., reduced to feet and inches, and added to 1 ft. 6.7 in. of the remainder, gives 12 rd. 4 yd. .7 in.

2. From 1⅝ bu. subtract ⅘ bu.

OPERATION.

1⅝ bu. = 1 bu.	2 pk.	4 qt.	0 pt.
⅘ bu. =	3	1	1⅕
	3	2	⅘

Or,

1⅝ bu. = $\frac{13}{8}$ bu.; $\frac{13}{8}$ bu. − ⅘ bu. = $\frac{33}{40}$ bu.

$\frac{33}{40}$ bu. = 3 pk. 2 qt. ⅘ pt.

ANALYSIS. — First find the value of each denominate fraction in integers of lower denominations (**433**), and subtract the less value from the greater.

Or, reduce the given fractions to fractions of the same denomination (**434**), then subtract the less from the greater, and find the value of their difference in integers of lower denominations.

3. From a pile of wood containing 42 Cd. 5 cd. ft., take 16 Cd. 6 cd. ft. 12 cu. ft., and how much remains?

4. From the sum of $\frac{2}{7}$ of $3\frac{3}{4}$ mi. and $17\frac{1}{7}$ rd., take $120\frac{1}{3}$ rd.

Find the difference between

5. $8\frac{9}{10}$ cwt. and $48\frac{3}{8}$ lb.
6. £$\frac{5}{9}$ and $\frac{2}{3}$ of $\frac{3}{4}$s.
7. $\frac{3}{40}$ lb. and 5 lb. 4 oz. 8 pwt.
8. .659 wk. and 2 wk. $3\frac{5}{6}$ da.
9. $\frac{117}{224}$ hhd. and 3.625 gal.
10. $\frac{2}{3}$ lea. and $\frac{7}{10}$ mi.
11. $\frac{5}{8}$ gross and $\frac{2}{3}$ doz.
12. ʒ 2 ℈ 1 and ℥ 4.
13. .9 da. and $\frac{1}{4}$ wk.
14. $\frac{15}{16}$ A. and 84.56 P.

15. If from a hhd. of molasses 14 gal. 1 qt. 1 pt. be drawn at one time, 10 gal. 3 qt. at another, and 29 gal. 1 pt. at another, how much will remain?

16. Of a farm containing 250 A., two lots were reserved, one containing 75 A. 136.4 P., and the other 56 A. 123.3 P.; the remainder was sold at \$$62\frac{1}{4}$ an acre. What did it sell for?

17. From 1 T. 11 cwt. 30 lb., take $\frac{4}{5}$ of a long ton.

18. From a pile of wood containing $125\frac{3}{4}$ Cd., was sold at one time 26 Cd. 7 cd. ft.; at another, 30 Cd. $4\frac{5}{8}$ cd. ft.; at another, $37\frac{9}{16}$ Cd. How much remained?

442. To find the interval of time between two dates.

1. How many yr., mo., da., and hr., from 3 o'clock P. M. of May 16, 1864, to 9 o'clock A. M. of Sept. 25, 1875?

OPERATION.

yr.	mo.	da.	hr.
1875	9	25	9
1864	5	16	15
11	4	8	18

ANALYSIS.—Since the later date expresses the greater period of time, write it as the minuend, and the earlier date as the subtrahend, writing the denominations in the order of the scale, then subtract.

1. When hours are to be obtained, reckon from 12 at night, and if minutes and seconds, write them still at the right of hours.

2. In finding the difference of time between two dates, 12 mo. are usually considered a year, and 30 days a month.

3. When the time is less than a year, the *true* number of days in each month and parts of a month is added.

4. The day on which a note, draft, or contract is *dated*, and that on which they *mature*, are *not both* included. The *former* is generally omitted.

2. The war between England and America was commenced April 19, 1775, and peace was restored Jan. 20, 1783. What length of time did the war continue?

3. The American Civil War began April 11, 1861, and closed April 9, 1865. What time did it continue?

4. How long has a note to run that is dated Jan. 16, 1873, and made payable July 10, 1875?

5. A note dated May 28, 1875, was paid Feb. 10, 1876. What length of time did it run?

6. A person started on a tour round the world at 9 o'clock A.M., Sept. 3, 1874, and returned to the same depot at 3 P.M., July 15, 1876. What time was he absent?

7. How many years, months, and days from your birthday to this date; or, what is your age?

8. How many days from June 20th to the 10th of Jan. following?

9. What length of time elapsed from 12 o'clock M., Jan. 10, 1876, to June 16, 9 o'clock A.M.?

10. What length of time elapsed from 16 min. past 10 o'clock A.M., July 4, 1873, to 22 min. before 8 o'clock P.M., Dec. 12, 1875?

11. What length of time will elapse from 40 min. 25 sec. past 12 o'clock M., April 21, 1875, to 4 min. 36 sec. before 5 o'clock A.M., Jan. 1, 1878?

MULTIPLICATION.

WRITTEN EXERCISES.

443. 1. Multiply 28 rd. 2 yd. 2 ft. by 7.

OPERATION.

28 rd.	2 yd.	2 ft.	
		7	
199	1 (½)	2	
	½ =	1	6 in.
199	2	0	6

ANALYSIS.—Write the multiplier under the lowest denomination of the multiplicand, and multiply.

7 times 2 ft. are 14 ft., equal to 4 yd. 2 ft. Write the 2 ft. under the feet, and reserve the 4 yd. to add to the product of yards.

7 times 2 yd. are 14 yd., and 4 yd. added make 18 yd., equal to 3 rd. 1½ yd. Write the 1½ yd. under the yards, and reserve the 3 rd. to add to the product of rods.

7 times 28 rd. are 196 rd., and 3 rd. added make 199 rd., which write under the same denomination.

The ½ yd. is equal to 1 ft. 6 in., which added to the product, gives 199 yd. 2 rd. 6 in. for the entire product.

1. The multiplier must be an abstract number. (**103.**)

2. When the multiplier is large and is a composite number, the work may be shortened by multiplying successively by its factors. (**109.**)

2. In 9 bbl. of walnuts, each containing 2 bu. 3 pk. 6 qt., how many bushels?

3. If a man cut 3 Cd. 36 cu. ft. of wood in 1 da., how many cords can he cut in 12 days?

4. Multiply 8 gal. 3 qt. 1 pt. 3.25 gi. by 96.

5. If 1 A. produce 42 bu. 1 pk. 5 qt. 1 pt. of corn, how many bushels will 64 A. produce?

6. Multiply O. 8 f℥ 9 fʒ 6 ♏ 34 by 24.

7. What will 84 yd. of cloth cost, at £1 8s. 9½d. a yd.?

8. If $80 will buy 3 A. 24 P. 20 sq. yd. 4 sq. ft. of land, *how much* will $4800 buy?

9. How many bushels of grain in 47 bags, each containing 2 bu. 2 pk. 6 qt.?

OPERATION.

$47 = (9 \times 5) + 2$

2 bu.	2 pk.	6 qt.		
		9		
24 bu.	0 pk.	6 qt.	in 9	bags.
		5		
120 bu.	3 pk.	6 qt.	" 45	"
5	1	4	" 2	"
126 bu.	1 pk.	2 qt.	" 47	"

ANALYSIS.—Multiplying the contents of 1 bag by 9, and the resulting product by 5, gives the contents of 45 bags, which is the composite number *next less* than the given prime number, 47. Next find the contents of 2 bags, which, added to the contents of 45 bags, gives the contents of 45 + 2, or 47 bags.

10. If a load of coal by the long ton weigh 1 T. 6 cwt. 2 qr. 26 lb. 10 oz., what will be the weight of 67 loads?

11. Multiply 4 yd. 1 ft. 4.7 in. by 125.

12. Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

13. At $1.37½ a gallon, what will be the cost of 5 casks of wine, each containing 28 gal. 2 qt. 1 pt.?

14. A farmer sold 4 loads of oats, averaging 41 bu. 3 pk. each, at $.75 a bushel. What did he receive for the whole?

DIVISION.

444. 1. Divide 56 lb. 9 oz. 12 pwt. by 6.

OPERATION.

	lb.	oz.	pwt.
6)	56	9	12
	9	5	12

ANALYSIS.—Write the divisor at the left of the dividend. The object is to find 1 *sixth* of a compound number.

$\frac{1}{6}$ of 56 lb. is 9 lb. and a remainder of 2 lb. Write the 9 lb. in the quotient, and reduce the 2 lb. to ounces, which, added to 9 oz., make 33 oz.

$\frac{1}{6}$ of 33 oz. is 5 oz. and a remainder of 3 oz. Write the 5 oz. in the quotient, and reduce the 3 oz. to pwt., which added to 12 pwt., make 72 pwt. $\frac{1}{6}$ of 72 pwt. is 12 pwt., which write in the quotient.

2. Divide 358 A. 57 P. 6 sq. yd. 2 sq. ft. by 7.

3. Divide £35 9s. 7d. by 5; by 7; by 8.

4. Divide 282 bu. 3 pk. 1 qt. 1 pt. by 9; by 10; by 12.

When the divisor is large, and is a *composite* number, the work may be shortened by dividing successively by its factors.

5. Divide 254 yd. 4 ft. 3⅓ in. by 21; by 42.

6. Divide 196 Cd. 4 cd. ft. 12 cu. ft. by 72.

7. How many iron rails, each 16 ft. long, are required to lay a railroad track 26 mi. long?

8. Divide 24 sq. mi. 140 P., by 22½.

9. Divide 202 yd. 1 ft. 6¾ in. by ⅗.

10. Divide 336 bu. 3 pk. 4 qt. by 4 bu. 3 pk. 2 qt.

Reduce both dividend and divisor to the same denomination, and divide as in simple numbers.

11. How many boxes, each holding 1 bu. 1 pk. 7 qt., can be filled from 356 bu. 3 pk. 5 qt. of cranberries?

12. Divide 311 gal. 1 qt. 1 pt. by 53.

OPERATION.

```
53)311 gal. 1 qt. 1 pt.(5 gal. 3 qt. 1 pt.
   265
   ---
    46 gal. rem.                 26 qt. rem.
     4                            2
   ---                           --
   185 qt. in 46 gal. 1 qt.      53 pt. in 26 qt. 1 pt.
   159                           53
   ---                           --
    26 qt. rem.
```

13. The aggregate weight of 41 hhd. of sugar is 19 T. 6 cwt. 22 lb. What is the average weight?

14. If a town 4 mi. square be equally divided into 62 farms, how much land will each farm contain?

LONGITUDE AND TIME.

445. The ***Longitude*** of a place is its distance east or west from a given meridian, measured on the equator.

The meridian from which longitude is reckoned is called the *first meridian*, and is marked 0°. All places east of this, within 180°, are in east longitude, and all places west, within the same limits, are in west longitude.

The English and Americans usually reckon longitude from the meridian of Greenwich, England; the French, from Paris.

446. Since the earth revolves on its axis once in 24 hours, the sun *appears to pass* from east to west around the earth, or through 360° of *longitude* once in 24 hours of time. Hence in 1 hour the sun appears to pass through $\frac{1}{24}$ of 360°, or 15°; in 1 minute, through $\frac{1}{60}$ of 15°, or 15′; and in 1 second, through $\frac{1}{60}$ of 15′, or 15″.

COMPARISON OF LONGITUDE AND TIME.

For a difference of	There is a difference of
15° in Long.	1 hr. in Time.
15′ " "	1 min. " "
15″ " "	1 sec. " "
1° " "	4 min. " "
1′ " "	4 sec. " "
1″ " "	$\frac{1}{15}$ sec. " "

1. Since the sun appears to move from east to west, when it is 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, knowing the difference of time between two places, and the exact time at one of them, the exact time at the other is found by *adding* their difference to the given time, if it is *east*, and by *subtracting*, if it is *west*.

2. If one place is in east and the other in west longitude, the difference of longitude is found by *adding* them, and if the sum is greater than 180°, by *subtracting* it from 360°.

ORAL EXERCISES.

447. 1. The earth revolves on its axis once in every 24 hr. What part of a revolution does it make in 12 hr.?

2. How many degrees of the earth's surface pass under the sun's rays in 24 hr.? In 12 hr.? In 4 hr.? In 1 hr.?

3. How many degrees of longitude cause a difference of 1 hr. in time? 2 hr.? 3 hr.?

4. When it is 6 o'clock at Chicago, what is the hour 15° east of Chicago? 15° west of Chicago?

5. When it is noon in New York, what is the hour 15° east of New York? 30° west of N. Y.?

6. When it is 3 o'clock at Washington, what is the time 15° 15′ east of Washington? 30° 30′ west?

7. What difference of longitude causes a difference of 1 hr. in time? Of 1 minute? Of 1 second?

8. If the difference in the time of Boston and of St. Louis is 1 hr. 15 min., what is the difference in their longitude?

9. A man left New Orleans and traveled east until his watch was 1 hr. 2 min. too fast. How far had he traveled, and in what direction?

10. Two persons, at different points, observe an eclipse of the moon; one seeing it at 9½ P. M., and the other at midnight. What is the difference in their longitude?

11. A tourist leaves home at 12 M. on Monday, and on Saturday finds his watch 1 hr. 15 min. slow. In what direction has he been traveling? How far?

12. A and B start from opposite points and travel towards each other. When they meet, A's watch is 40 min. slow and B's 1 hr. fast. How far apart are the two points of starting, and in what direction did each travel?

WRITTEN EXERCISES.

448. To find the difference of longitude between two places, when the difference of time is known.

1. When it is 9 o'clock at Washington, it is 7 min. 4 sec. past 8 o'clock at St. Louis. Find the diff. of longitude.

OPERATION.

9 hr.	0 min.	0 sec.	
8	7	4	
	52	56	Diff. in Time.
		15	
13°	14′	00″	Diff. in Long.

Or, 4) 52 min. 56 sec.
13° 14′

ANALYSIS.—Since every hour of time corresponds to 15° of long., and every minute of time to 15′ of long., and every second of time to 15″ of long. (**446**), there are 15 times as many deg., min., and sec. in the difference of longitude, as there are hr., min., and sec. in the difference of time. Or,

Since 4 min. of time make a difference of 1° of long., and 4 sec. of time a difference of 1′ of long., there will be ¼ as many degrees of long. as there are minutes of time, and ¼ as many minutes of long. as there are seconds of time.

2. The difference in the time of Washington and of St. Petersburgh is 7 hr. 9 min. 19¼ sec. What is the difference in their longitudes?

3. When it is 12 o'clock M. at Rochester, N. Y., it is 9 hr. 1 min. 37 sec. A. M. at San Francisco. The long. of Rochester being 77° 51′ W., what is the long. of the latter?

RULE.—*Multiply the difference of time expressed in hours, minutes, and seconds by* 15 ; *the product will be the difference of longitude in degrees, minutes, and seconds.* Or,

Reduce the difference of time to minutes and seconds, then divide by 4 ; *the quotient will be the difference of longitude in degrees and minutes.*

4. Noon comes 1 hr. 5 min. 42 sec. sooner at Quebec than at Chicago, whose longitude is 87° 37' 45". What is the longitude of Quebec?

5. When the days and nights are of equal length, and it is noon on the first meridian, on what meridian is it then sunrise? Sunset? Midnight?

449. *The following table of the Longitude of places is compiled from the Records of the U. S. Coast Survey.*

Albany	73° 44' 50" W.	New York	74° 3' W.
Ann Arbor	83° 43' W.	New Orleans	90° 2'.30" W.
Astoria, Or.	124° W.	Paris	2° 20' E.
Boston	71° 3' 30" W.	Rome	12° 27' E.
Berlin	13° 23' 45" E.	Richmond, Va.	77° 25' 45" W.
Bombay	72° 54' E.	San Francisco	122° 26' 45" W.
Cincinnati	84° 29' 31" W.	St. Paul, Minn.	95° 4' 55" W.
Chicago	87° 37' 45" W.	St. Louis, Mo.	90° 15' 15" W.
Cambridge, Mass.	71° 7' 40" W.	Univ. of Virginia	78° 31' 30" W.
Jefferson City, Mo.	92° 8' W.	West Point, N. Y.	73° 57' W.
Mexico	99° 5' W.	Washington, D. C.	77° 0' 15" W.

450. To find the difference of time between two places, when their longitudes are given.

1. Find the diff. in the time of Cinn. and of St. Paul.

OPERATION.

$$\begin{array}{rrrrl} & 95° & 4' & 55'' & \text{Long of St. P.} \\ & 84 & 29 & 31 & \text{“ Cinn.} \\ \hline 15) & 10° & 35' & 24'' & \text{Diff. of Long.} \\ \hline & & \text{42 min.} & 21\frac{3}{5}\text{ sec.} & \text{Diff. of Time.} \end{array}$$

ANALYSIS.—Since 15° of longitude make a difference of 1 hr. of time, and 15', a difference of 1 min. of time, and 15", a difference of 1 sec. of time (**446**), there are $\frac{1}{15}$ as many hours, minutes, and seconds of time as there are degrees, minutes, and seconds of longitude.

Or,

$$\begin{array}{llllll} 10^\circ & 35' & 24'' & & & \\ & & 4 & & \text{min.} & \text{sec.} \\ \hline 42 \text{ min.} & 21 \text{ sec.} & \frac{36}{60} & = & 42 & 21\frac{3}{5} \end{array}$$

Since 1° of long. makes a diff. of 4 min. of time, and 1′ makes a diff. of 4 sec. of time (**446**), there is a diff. of 4 times as many minutes and seconds of time as there are deg., min., and sec. of long.

2. Find the difference in the time of Ann Arbor, Mich., and of Cambridge, Mass?*

3. When it is half-past 3 o'clock P.M. at West Point, N. Y., what time is it at Bombay?

RULE.—*Divide the difference of longitude expressed in degrees, minutes, and seconds, by* 15; *the quotient will be the difference of time in hours, minutes, and seconds.* Or,

Multiply the difference of longitude by 4, *and the product will be the difference of time in minutes and seconds, which may be reduced to hours.*

Find the difference in the time of

4. Washington, and Rome.
5. Chicago, and Paris.
6. N. Orleans, and N. York.
7. Albany, and Jefferson C'y.
8. Richm'd, and St. Louis.
9. New York, and Mexico.
10. Ann Arbor, and Berlin.
11. Mexico, and San Fran.

12. When it is 6 A.M. at Boston, what time is it at Cincinnati? At Chicago? At St. Louis?

13. When it is 6 P.M. at the University of Va., what time is it at Berlin? At St. Paul? At Astoria, Or.?

14. How much later does the sun rise in New York than in Rome? Than in Paris?

15. In sailing from San Francisco to Bombay, will a chronometer gain or lose time, and how much?

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* **Take from the Table the required Longitude of the different places.**

DUODECIMALS.

451. ***Duodecimals*** are fractions of a foot formed by successively dividing by 12; as, $\frac{1}{12}$, $\frac{1}{144}$, $\frac{1}{1728}$, etc.

452. The ***Unit*** of measure is 1 *foot*, which may be a *linear*, a *square*, or a *cubic foot*. The *scale* is uniformly 12.

453. In the duodecimal divisions of a foot, the different orders of units are related as follows:

1′ (inch or prime)	= $\frac{1}{12}$ of a foot,	or 1 in.	Linear Meas.
1″ (second) or $\frac{1}{12}$ of $\frac{1}{12}$	= $\frac{1}{144}$ of a foot,	or 1 "	Square "
1‴ (third) or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$	= $\frac{1}{1728}$ of a foot,	or 1 "	Cubic "

TABLE.

12 Fourths (⁗)	=1 Third . .	1‴	1 *ft.*=12′=144″=1728‴=20736⁗
12 Thirds	=1 Second . .	1″	1′= 12″= 144‴= 1728⁗
12 Seconds	=1 Prime . .	1′	1″= 12‴= 144⁗
12 Primes	=1 Foot . . .	*ft.*	1‴= 12⁗

The marks ′, ″, ‴, ⁗, are called *Indices*.

Duodecimals are used by *artificers* in measuring *surfaces* and *solids*.

ADDITION AND SUBTRACTION.

454. Duodecimals are added and subtracted in the same manner as compound numbers.

WRITTEN EXERCISES.

1. Add 14 ft. 7′ 8″, 16 ft. 3′ 5″, and 21 ft. 9′ 11″.
2. Add 140 ft. 10′ 7″ 9‴, 71 ft. 8″, and 107 ft. 4′ 11″ 3‴.
3. From 54 ft. 9′ 5″ subtract 30 ft. 10′ 8″.

Duodecimals are not much used. The subject is fully treated and applied in "Robinson's Higher Arithmetic."

MULTIPLICATION.

455. In the multiplication of duodecimals, the product of *two* dimensions is *area* or *surface,* and the product of *three* dimensions is *solidity* or *volume.* **(344, 349.)**

WRITTEN EXERCISES.

456. 1. Multiply 9 ft. 8′ by 4 ft. 7′.

OPERATION.

9 ft.	8′	
4 ft.	7′	
5 ft.	7′	8″
38 ft.	8′	
44 ft.	3′	8″

ANALYSIS.—Begin at the right. 8′ × 7′ = 56″ = 4′ 8″. Write the 8″ one place to the right, reserving the 4′ to add to the next product.

Then 9 ft. × 7′ = 63′; 63′ + 4′ = 67′ = 5 ft. 7′, which write in the places of feet and primes.

Next multiply by 4 feet; 8′ × 4 ft. = 32′ = 2 ft. 8′. Write the 8′ in the place of primes, reserving the 2 ft. to add to the next product. Then 9 ft. × 4 ft. = 36 ft.; 36 ft. + 2 ft. = 38 ft., which write in the place of feet. Adding the partial products, the sum equals 44 ft. 3′ 8″, the product required.

2. How many square feet in 4 boards, each 12 ft. 9′ long, and 1 ft. 4′ wide?

RULE.—I. *Write the terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Multiply each term of the multiplicand by each term of the multiplier, beginning with the lowest order of units in each. Reduce each product to higher denominations when possible, and write in their proper places. The sum of the partial products will be the product required.*

3. Multiply 10 ft. 6′ 4″ by 5 ft. 3′ 8″.

4. Find the area of a floor 14 ft. 8′ wide and 16 ft. 5′ long.

5. What are the solid contents of a block of marble 6 ft. 10′ long, 4 ft. 3′ wide, and 1 ft. 9′ thick?

457. SYNOPSIS FOR REVIEW.

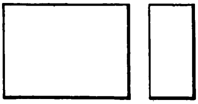
DENOMINATE NUMBERS—CONTINUED.

- 3. REDUCTION.
  - 1. Definition of Reduction.
  - 2. Reduction to Lower Denominations.
    - 1. Principle.
    - 2. Rule, I, II.
  - 3. Reduction to Higher Denominations.
    - 1. Principle.
    - 2. Rule, I, II.
  - 4. Reduction of Denominate Fractions.
    - **430.** Rule.
    - **432.** Rule, I, II. III.
    - **434.** Rule.
    - **436.** Rule, I, II, (2.)
- 4. ADDITION OF COMP. NUMBERS.
- 5. SUBTRACTION " "
- 6. MULTIPLICATION " "
- 7. DIVISION " "
  - 1. How performed.
  - 2. Upon what principles based.
- 8. LONGITUDE AND TIME.
  - 1. Definitions.
    - 1. Longitude.
    - 2. First Meridian.
  - 2. Comparison of Longitude and Time.
  - 3. Rules.
    - 1. To find diff. of long. when diff. of time is given.
    - 2. To find diff. of time when diff. of long. is given.
- 9. DUODECIMALS.
  - 1. Definition.
  - 2. Unit of Measure.
  - 3. Table.
  - 4. Addition and Subtraction.
  - 5. Multiplication.
    - 1. Product of two dimensions.
    - 2. Product of three dimensions.
    - 3. Rule, I, II.

MEASUREMENTS

458. ***Measurements*** involve a practical application of the Weights and Measures to various operations required in the mechanic arts, and to the common business of life.

RECTANGULAR SURFACES.*



Rectangle.

459. A ***Rectangle*** is a plane figure bounded by four sides, having all its angles right angles.

It has *two* dimensions—*length and breadth*.

When *all* its sides are equal, it is called a *Square*.

460. The ***Area*** of a rectangle is the surface included within the lines which bound it, and is expressed by the number of times it contains a given *unit of measure*.

461. The ***Unit of Measure*** for surfaces is a *square* each side of which is a unit of some known length.

Thus, the unit of square inches is 1 *square inch;* of square feet, 1 *square foot;* of square yards, 1 *square yard*, etc.

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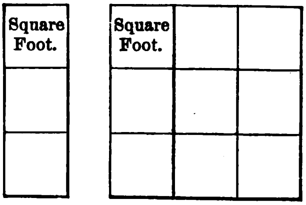
* Measurements of plane figures requiring a knowledge of Involution and Evolution are treated at the close of this book under the head of "*Mensuration*."



3 sq. ft. × 3 = 9 sq. ft.

The diagram represents a square yard, each side of which is 1 yd. or 3 ft. long, and the whole is divided into square feet, 1 sq. ft. being the *Unit of Measure.* In one row there are 3 sq. ft., in 3 rows there are 3 times 3 sq. ft., or 9 sq. ft. Hence the *area* of 1 sq. yd. is 9 sq. ft.

So the area of a *rectangle* formed by 2 adjacent rows, is expressed by 3 sq. ft. × 2, or 6 sq. ft.

462. To find the *area* of a rectangle :

RULE.—*Find the product of the numbers denoting the length and breadth, expressed in the same denomination ; the result is the area.*

463. To find either *dimension* of a rectangle :

RULE.—*Divide the area by one dimension ; the quotient is the other.*

464. ***Artificers*** compute their work by *linear, superficial* or *square,* and *cubic measures.*

1. Glazing and stone-cutting are estimated by the *square foot.*
2. Plastering, paving, ceiling, etc., commonly by the *square foot,* or the *square yard.*
3. Roofing, flooring, partitioning, slating, etc., generally by the *square* of 100 *square feet ;* sometimes by the *square foot,* or *square yard.*
4. *One thousand* shingles, averaging 4 in. wide, and laid 5 in. to the weather, are estimated to cover a *square.*
5. Bricklaying is generally estimated by the *thousand bricks.*

WRITTEN EXERCISES.

465. 1. How many square feet in a floor 27 ft. long and 21 ft. wide ? How many square yards ?

2. How many feet wide is a hall that is 26 ft. long and contains 195 square feet ?

3. What is the length of a lawn that contains 305 sq. yd. and is 45 ft. wide?

Find the area of rectangles having the following dimensions:

4. $12\frac{1}{3}$ yards square.
5. 18 yd. 2 ft. square.
6. $18\frac{1}{2}$ rd. by $20\frac{1}{2}$ rd.
7. 5 ch. 14 l. by 6 ch. 25 l.
8. 25 ft. 6 in. by 16 ft. 9 in.
9. 14 yd. 1 ft. 10 in. square.

The area and one dimension given, find the other dimension of the following rectangles:

10. Area $374\frac{1}{8}$ square feet, length 20 ft. 6 in.

11. Area 5 A. 41 P., width 7 chains 25 links.

12. Area 180 sq. yd. 4 sq. ft., width 9 yd. 2 ft.

13. How many square yards in the sides of a room 16 ft. long, 12 ft. 6 in. wide, and 9 ft. 3 in. high?

14. How many planks 12 ft. long and 10 in. wide will be required to floor a room which is 24 ft. by 20 ft.?

Find the number of yards in length of carpeting required for rooms of the following dimensions:

15. For a room 24 ft. by 16 ft. 6 in.; carpet 1 yd. wide.

16. For a room 52 ft. by 35 ft.; carpet 2 ft. 4 in. wide.

17. For a room 28 ft. by 23 ft. 9 in.; carpet 30 in. wide.

18. For a room 27 ft. 3 in. by 22 ft. 6 in.; carpet 2 ft. 6 in. wide.

Find the cost of carpeting rooms, their dimensions, and the width and price of carpet being as follows:

19. Floor, 34 ft. by 18 ft. 6 in., carpet 2 ft. wide, at \$.94 a yard.

20. Floor, 30 ft. 3 in. by 22 ft., carpet $\frac{3}{4}$ yd. wide, at \$1.08 a yard.

21. Floor, $18\frac{1}{2}$ ft. by 16.4 ft., carpet $\frac{7}{8}$ yd. wide, at \$$2\frac{1}{8}$ a yard.

22. Floor, 40 ft. by 36 ft., covered with matting 4 ft. wide, at $1.22 a yard.

23. Floor, 26 ft. 6 in. by 18 ft., covered with oil-cloth, at $1.15 a square yard.

24. How many tiles 8 inches square, will lay a floor 48 ft. by 10 ft.?

25. What will be the cost of flagging a side-walk 312 ft. long and 6½ ft. wide, at $2.70 a square yard?

26. What will it cost to cement a cellar bottom 48 ft. 6 in. long and 27 ft. wide, at $.45 a square yard?

27. How many squares are there in a partition 104 ft. 9 in. long, and 20 ft. 4 in. high?

28. What is the expense of plastering the sides and ceiling of a room 40 ft. long, 36½ ft. wide, and 22¼ ft. high, at $.36 a square yard, allowing 1375 sq. ft. for doors, windows, and baseboard?

29. Find the cost of glazing 6 windows, each 8 ft. 3′, by 5 ft. 4′, at $.75 a square foot.

30. A room is 24 ft. by 16½ ft., and 18 ft. high. Find the cost of papering its sides with paper 40 in. wide and 8 yd. in a roll, at $1.20 a roll put on, and edging it with gilt moulding next the ceiling, at 9 cents a foot. There are two windows, each 2 ft. 4 in. by 5 ft. 8 in., and 2 doors 2 ft. 9 in. by 6 ft. 6 in., and a baseboard 9 in. wide.

31. How many sods, each 16 in. square, will be required to turf a yard 53 ft. 4 in. long and 28 ft. wide?

32. How many yards of silk, ⅝ yd. wide, will be required to line 24 yd. of satin ¾ yd. wide?

33. How many rolls of paper, each 8 yd. long and 18 in. wide, will paper the sides of a room 16 ft. by 14 ft. and *10 ft. high,* deducting 124 sq. ft. for doors and windows?

34. Find the cost of lining a tank 5 ft. 8 in. long, 4 ft. wide, and 5 ft. deep, with zinc, weighing 5 lb. to the square foot, at 12 cents a pound, which includes the labor.

35. Find the cost of plastering the walls of a room 12 ft. 11′ square, 9 ft. 3′ high, allowing for 2 windows and 1 door, each 6 ft. 2′ by 2 ft. 4′, at $.28 a square yard.

36. How many shingles 4 in. wide, laid 6 in. to the weather, will cover the roof of a building, the ridge being 46 ft. long, and the girt from eaves to eaves 40 ft., the first course on each of the eaves being double?

37. What will be the cost of wainscoting a room 21 ft. 8 in. by 14 ft. 10 in. and 10 ft. 6 in. high, at $.30 a sq. yd.?

38. Find the cost of slating a roof 64 ft. 9 in. long and 45 ft. wide, at $15.37½ per square?

LAND.

466. The ***Unit*** of land measure is the *acre*.

Measurements of land are commonly recorded in *square miles*, *acres*, and *hundredths* of an acre. The denomination *rood* is no longer used. See Arts. **341** and **346**.

WRITTEN EXERCISES.

467. 1. How many acres in a farm 120 rods square?

2. A field 80 rd. long contains 16 A.; what is its width?

3. A town 6½ mi. long and 5½ mi. wide is equal to how many farms of 120 A. each?

4. What decimal part of an acre is a piece of land 121 yd. long and 75 feet wide?

5. A rectangular farm containing 435 A. 96 P. is 264 rd. long on one side: what is the length of the other side?

6. What is the value of a farm 189.5 rd. long and 150 rd. wide, at $42¾ an acre?

7. A man having a field 70 rd. square appropriated 5 A. of it to corn, 100 sq. rd. to garden vegetables, and the remainder to meadow. What fractional part of the whole field did the meadow comprise?

8. A rectangular field 50 rd. long contains 10 acres; Another field of the same width contains 5 acres; what is its length?

9. At $2.75 a rod, how much less will it cost to fence a piece of land 80 rd. square than if the same were in the form of a rectangle twice as long and one-half as wide?

10. I bought a piece of land 16 ch. long and 15 ch. wide, at $100 an acre, and dividing it into lots of 6 rd. by 5 rd., sold them at $50 each. What was my gain?

468. ***Government Lands*** are usually surveyed into rectangular tracts, bounded by lines conforming to the cardinal points of the compass.

A *Base-line* on a parallel of latitude, and a *Principal Meridian* intersecting it, are first established. Other lines are then run *six miles* apart, each way, as nearly as possible.

The tracts thus formed are called *Townships*, and contain, as near as may be, 23040 acres.

A line of townships extending north and south is called a *Range*.

The *ranges* are designated by their number east or west of the *principal meridian*.

The *townships* in each range are designated by their number north or south of the *base-line*.

Since the earth's surface is convex, the principal meridians converge as they proceed northward. This tends to throw the townships and sections out of square, and necessitates occasional lines of offset, called "*correction lines.*"

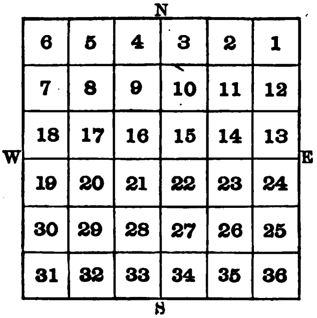
Townships are subdivided into *Sections,* and sections into *Half-Sections, Quarter-Sections, Half-Quarter-Sections, Quarter-Quarter-Sections,* and *Lots.*

Diagram No. 1 shows the sub-divisions of a *Township* into Sections, and how they are *numbered,* commencing at the N.E. corner.

Diagram No. 2 shows the sub-divisions of a *Section,* on an enlarged scale, and how they are *named.*

Diagram No. 1.

A Township.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Diagram No. 2.

A Section.

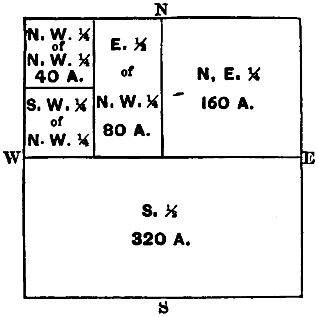


TABLE.

6 mi. × 6 mi. = 36 sq. mi. = 23040 Acres. = 1 Township.
1 " × 1 " = 1 " = 640 " = 1 Section.
1 " × $\frac{1}{2}$ " = $\frac{1}{2}$ " = 320 " = 1 Half-Section.
$\frac{1}{2}$ " × $\frac{1}{2}$ " = $\frac{1}{4}$ " = 160 " = 1 Quarter-Section.
$\frac{1}{2}$ " × $\frac{1}{4}$ " = $\frac{1}{8}$ " = 80 " = 1 Half-Quarter-Section.
$\frac{1}{4}$ " × $\frac{1}{4}$ " = $\frac{1}{16}$ " = 40 " = 1 Quarter-Quarter-Section.

A *Lot* is a sub-division of a section, usually of irregular form, on account of bordering upon a navigable river or lake—containing as near as may be the area of a Quarter-Quarter-Section, and described as lot No. 1, 2, 3, etc., of a particular section.

City and village plats are usually sub-divided into *Blocks,* and these into *Lots.*

WRITTEN EXERCISES.

1. If a township of land is equally divided among 288 families, how many acres does each receive? What part of a section?

2. What number of rails will enclose a quarter section of land with a fence 6 rails high, and 3 lengths for every 2 rods; and what will be the cost of the rails, at $40 per thousand?

3. A man bought the S. ½ of a section of land at $2¼ an acre, and afterward sold the E. ½ of what he bought at $4.37½ an acre. What was his gain?

4. If I buy the N.E. ¼ and the E. ½ of N.W. ¼ of a section of land, how many acres do I purchase? What part of a whole section? How are the parts located in respect to each other?

5. A speculator bought of the Ill. Central R. R. Co., the S. ½ of Section 4, township 10 north, range 6 east, at $2 an acre. He afterward sold the E. ½ of S.E. ¼ at $2.75 an acre; the N.W. ¼ of S.E. ¼ at $3½ an acre; and the N. ½ of S.W. ¼ at $3.84 an acre. How many acres has he left? What was his gain on the purchase price of the whole? Draw diagram.

6. A man having purchased a section of land from the U. S. Government at $1.25 an acre, sold the S. ½ of S.W. ¼ at $2.50 an acre; the N.E. ¼ of N.W. ¼ at $1.75 an acre; the W. ½ of S.E. ¼ at $2 an acre; and the W. ½ of S.W. ¼ of N.E. ¼ at $3 an acre. How many acres has he remaining, and what is his gain, provided the remainder is sold at $2¼ an acre? Draw diagram and explain.

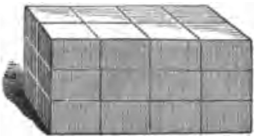
RECTANGULAR SOLIDS.

469. A ***Rectangular Solid*** is a body bounded by six rectangular plane *faces.*

The opposite sides are equal and parallel.

It has *three* dimensions—*length, breadth,* and *thickness.*

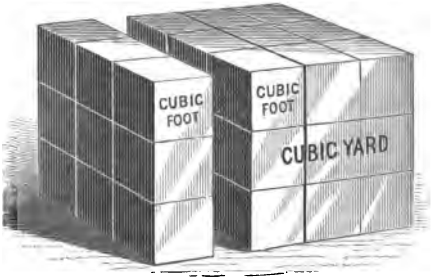
When *all* its faces are equal, it is called a *Cube.*



470. The ***Volume,*** or ***Solid Contents*** of a body is the space included within the surfaces which bound it, and is expressed by the number of times it contains a given *unit of measure.*

471. The ***Unit of Measure*** for solids is a *cube,* the edge of which is a unit of some known length.

Thus, the unit of cubic inches is a *cube* the edge of which is 1 *inch,* or 1 *cubic inch;* of cubic feet, 1 *cubic foot,* etc.



9 cu. ft. × 3 = 27 cu. ft.

The diagram represents a cubic yard, each face being a square yard, containing 9 sq. ft. If a section 1 ft. thick is cut from one side, it may be divided into 3 times 3 cu. ft., or 9 cu. ft., 1 cu. ft. being the *unit of measure.* And since the cubic yard is 3 ft. thick, it contains 3 such sections, or 3 times 9 cu. ft., which are 27 cu. ft. Hence the *volume* of 1 cu. yd. is 27 cu. ft.

So the volume of a *solid,* formed of two adjacent sections, is expressed by 3 cu. ft. × 3 × 2 = 18 cu. ft.

472. To find the *volume* of a rectangular solid :

RULE.—*Find the product of the numbers denoting the three dimensions, expressed in the same denomination; this result is the volume.*

473. To find a *required dimension* of a rectangular solid :

RULE.—*Divide the volume by the product of the numbers denoting the other two dimensions; the quotient will be the required dimension.*

WRITTEN EXERCISES.

474. 1. What are the contents of a rectangular solid 6 ft. long and 4 ft. square ?

2. What is the volume of a solid 9 ft. long, 4 ft. wide, and 3 ft. thick ?

3. A vat 12 ft. square contains 1224 cu. ft. What is its depth ?

4. What is the volume of a bin, the inside dimensions of which are 8 ft. 6 in. by 6 ft. by 4 ft. 4 in.

5. How many cubic yards of earth must be removed in digging a cellar 36 ft. long, 24 ft. wide, and 6½ ft. deep ?

Find the volume of rectangular solids having the following dimensions :

6. Of a cube the edge of which is 1 yd. 1 ft. 9 in.

7. Of a solid 6 yd. 2 ft. 7 in. by 3 ft. 4 in. by 2 ft. 11 in.

8. Of a solid 5 ft. square and the height 6.4 ft.

Find the required dimension of rectangular solids, the volumes and two dimensions being as follows :

9. Volume, 6 cu. ft. ; length, 8 ft. ; width, 8 ft.

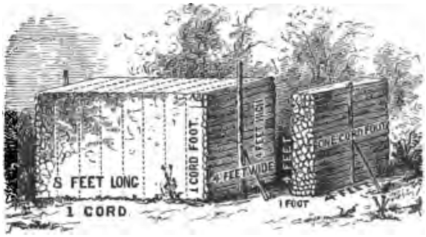
10. Volume, 20 cu. ft.; length, 36 ft.; width, 10 in.

11. Volume, 13 cu. yd. 14 cu. ft. 900 cu. in.; width, 7 ft. 3 in.; height, 5 ft. 6 in.

12. How many cubic feet of air in a room that is 24 ft. 9 in. long, 18 ft. 4 in. wide, and 10 ft. 8 in. high?

A *Cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

A *cord-foot* is 1 foot in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.



13. How many cords in a pile of wood 30 ft. long, 8 ft. wide, and 6 ft. 6 in. high?

14. A pile of wood containing 67½ cords, is 90 ft. long and 12 ft. wide. How high is it?

15. What will be the cost of a pile of wood 12 ft. 6 in. long, 8 ft. wide, and 4 ft. 6 in. high, at $3.75 a cord?

16. What will it cost to dig a cellar 45 ft. long, 28 ft. wide, and 8 ft. 6 in. deep, at $.42 a cubic yard?

17. What must be the length of a load of wood that is 3 ft. high and 5 ft. 4 in. wide, to contain a cord?

18. How many cans, 8 in. by 6 in. by 3 in., can be packed in a box 36 in. by 20 in. by 15 in. in the clear?

19. At $3½ a cord, what is the value of the wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

20. In building a house, 200 joists 10 in. by 3 in. were used, which together amounted to 1000 cu. ft. What was the length of each?

475. ***Masonry*** is estimated by the *cubic foot,* and by the *perch;* also by the *square foot* and the *square yard.*

1. *Materials* are usually estimated by cubic measure; the *work* by cubic or square measure.

2. A *Perch* of stone, or of masonry, is 16½ ft. long, 1½ ft. wide, and 1 ft. high, and is equal to 24.75 cu. ft.

3. When stone is built into a wall, 22 cu. ft. make a perch, 2¾ cu. ft. being allowed for mortar and filling.

4. *Embankments* and *Excavations* are estimated by the *cubic yard.*

5. A cubic yard of common earth is called a *load.*

6. *Brickwork* is generally estimated by the *thousand bricks;* sometimes in *cubic feet.* In *walls,* brick-work is estimated at the rate of a brick and a half thick.

7. North River bricks are 8 in. × 3½ × 2¼; Maine bricks are 7½ in. × 3⅜ × 2⅜; Philadelphia and Baltimore bricks are 8¼ in. × 4⅛ × 2⅜; and Milwaukee bricks 8½ in. × 4⅛ × 2⅜.

8. In estimating *material,* allowance is made for doors, windows, and cornices.

9. In estimating the *work,* masons measure each wall on the *outside,* and ordinarily, no allowance is made for doors, windows, and corners; but sometimes an allowance of *one-half* is made, this being, however, a matter of *contract.*

476. To find the number of bricks in a cubic foot of masonry:

RULE.—I. *Add to the face dimensions of the kind of bricks used one-half the thickness of the mortar or cement in which they are laid, and compute the area.*

II. *Multiply this area by the quotient of the thickness of the wall divided by the number of bricks of which it is composed, the product will be the volume of a brick and its mortar in cubic inches.*

III. *Divide* 1728 *by this volume, and the quotient will be the number of bricks in a cubic foot.*

WRITTEN EXERCISES.

477. 1. How many Milwaukee bricks in a cubic foot of wall 12¾ in. wide, laid in courses of mortar ¼ of an inch thick?

OPERATION.

$8.5 + (\overline{.25 \times 2} \div 2) = 8.75$ in. = *length* of brick and joint.

$2.375 + (\overline{.25 \times 2} \div 2) = 2.625$ in. = *thickness* of brick and joint.

$8.75 \times 2.625 = 22.96875$ sq. in. = *area* of its face.

$12.75 \div 3$ (number of bricks in width of wall) $= 4.25$ in. = *width* of brick and mortar.

$22.96875 \times 4.25 = 97.617+$ = *cubic inches* in a brick.

$1728 \div 97.617+ = 17.7+$ = *number of bricks* in a cubic foot.

2. How many bricks, 8 in. × 4 × 2, will be required to build a wall 42 ft. long, 24 ft. high, and 16½ in. thick, laid in courses of mortar ¼ of an inch thick?

3. How many perches of stone, laid dry, will build a wall 60 ft. long, 16½ ft. high, and 18 in. thick?

RULES.—1. *Multiply the number of cubic feet in the wall, or work to be done, by the number of bricks in a cubic foot; the product will be the number of bricks required.*

2. *Divide the number of cubic feet in the work to be done by 24.75; the quotient will be the number of perches.*

4. How many perches of masonry in a wall 120 ft. long, 6 ft. 9 in. high, and 18 in. thick?

5. How many bricks, 8½ in. × 4 × 2¼, will be required to build the four walls of a house 36 ft. long, 24 ft. high, and 12⅜ in. thick, allowing 224 cu. ft. for doors and windows, and ¼ of an inch for each course of mortar?

6. At $.56 a cu. yd., what will it cost to remove an embankment 240 ft. long, 38 ft. wide, and 8.5 ft. high?

7. Find the cost of digging and walling the cellar of a house, whose length is 41 ft. 3 in., and width 33 ft.; the cellar to be 8 ft. deep, and the wall 1½ ft. thick. The excavating will cost $.50 a load, and the stone and mason work $3.75 a perch.

8. How many perches of stone will be required to enclose a lot 16 rd. long and 12 rd. wide, with a wall 6 ft. high and 3 ft. thick?

9. A street 650 ft. long and 72 ft. wide, averages 4.5 ft. below grade. Find the cost of filling it in, at $.42 a cu. yd.?

10. What will be the cost of building a wall 60 ft. long, 21⅓ ft. high, and 17 in. thick, of Philadelphia bricks, laid in courses of mortar ¼ of an inch thick, at $12½ per M.?

11. How many cubic feet of masonry in the wall of a cellar 37½ ft. long, 26 ft. wide, and 9 ft. deep, the wall being 2 ft. thick, allowing *one-half* for the corners; and what will be the cost, at $3.85 a perch?

BOARDS AND TIMBER.

478. A ***Board Foot*** is 1 ft. long, 1 ft. wide, and 1 inch thick. Hence 12 board feet make 1 cubic foot.

Board feet are changed to cu. ft. by dividing by 12, and cubic feet are changed to board feet by multiplying by 12.

1. In *Board Measure* all boards are assumed to be 1 in. thick.
2. *Lumber* and *Sawed Timber*, such as plank, scantling, joists, etc., are usually estimated in *board* measure.
3. *Hewn* and *Round Timber* are commonly estimated in *cubic* measure.

479. When lumber is not more than 1 inch thick:

RULE.—*Multiply the length in feet by the width in inches, and divide the product by* 12.

480. When more than 1 inch thick :

Rule.—*Multiply the length in feet by the width and thickness in inches, and divide the product by* 12.

1. If *one* of the dimensions is *inches*, and the other two are *feet*, the product will be *board feet.*

2. If a board tapers regularly, multiply the length by the *mean width*, found by taking half the sum of the two ends.

WRITTEN EXERCISES.

481. 1. Find the contents of a board 15 ft. long and 8 in. wide.

Operation.—$\overline{15 \times 8} \div 12 = 10$ board feet.

2. What are the contents in board measure of a joist 16 ft. long, 10 in. wide, and 3 in. thick ?

Operation.—$\overline{3 \times 10 \times 16} \div 12 = 40$ board feet.

3. How many board feet in 4 boards 16 ft. long, 10 in. wide ?

4. How many board feet in 2 joists 17 ft. long, 11 in. wide, and 3 in. thick ?

5. Find the contents of a board 18 ft. long, 1 ft. 8 in. wide at one end, and 14 in. at the other.

Operation.—$\overline{20 \text{ in.} + 14 \text{ in.}} \div 2 = 17$ in. ; $\overline{18 \times 17} \div 12 = 25\frac{1}{2}$ board ft.

6. Find the cost of 5 boards 12 ft. long, 17 in. wide at one end and 11 in. at the other, at 6 cents a square foot.

7. Find the cost of 10 planks, each 15 ft. long, 16 in. wide, and $3\frac{1}{2}$ in. thick, at $2.25 per hundred feet.

8. What length of board 9 in. wide contains 8 board ft.?

Operation.—$\overline{144 \times 8} \div 9 = 128$; $128 \div 12 = 10\frac{2}{3}$ ft., the length.

9. What length may be cut from a board 15 ft. long and 20 in. wide, so as to leave 15 board feet ?

10. What must be the width of a board 16 ft. long to contain 12 board feet?

OPERATION.—16 ft. = 192 in.; $\overline{144 \times 12} \div 192 = 9$ in., the width.

11. What must be the width of a piece of board 5 ft. 3 in. long, to contain 7 square feet?

12. Find the cost of 3 pieces of timber, each 26 ft. long and 6 in. by 9 in., at $1.75 per hundred board feet.

13. Find the cost of 8 pieces of scantling, 3 in. by 4 in. and 14 ft. long, at $9.50 per thousand board feet.

14. What length of a piece of timber 6 in. by 9 in., will contain 3 cubic feet?

OPERATION.—$\overline{1728 \times 3} \div \overline{9 \times 6} = 96$; $96 \div 12 = 8$ ft., the length.

15. A piece of timber is 10 in. by 16 in.; what length of it will contain 15 cubic feet?

16. What amount of inch lumber will make a box 4 ft. by 3 ft. 6 in. by 2 ft. 6 in. on the outside?

Find the cost of the following:

17. Of 36 boards, 12 ft. long, 11 in. wide, @ $2¼ per C.

18. Of 16 planks, 14½ feet long, 10 in. wide, and 3 in. thick, @ $16¼ per M.

19. How many board feet in a stick of timber 36 ft. long, 10 in. thick, 12 in. wide at one end and 9 in. wide at the other end? How many cubic feet?

20. Make a bill for lumber bought by John Osborn of Geo. Mason & Co., of St. Paul, Sept. 20, 1875, as follows:

124 boards,	10 in.	by 16 ft.	@ $15	per M.
120 "	16 "	" 14 "	" $16½	"
40 planks,	2½ × 12	" 15 "	" $18.75	"
96 joists,	3 × 10	" 18 "	" $14	"
60 scantling,	3 × 4	" 12 "	" $12½	"

What is the amount?

21. Find the cost of the flooring for a two-story house at $30 per M., it being 1¼ in. thick, each floor being 48 ft. by 25 ft., no allowance made for waste.

22. A rectangular field, 16 ch. long and 8 ch. wide, is enclosed by a post and board fence; the posts are set 8 ft. apart, the boards are 16 ft. long, and the fence is 5 boards high. The bottom board is 12 in. wide, the top board 6 in., and the other three each 9 in. wide. The posts cost $25 per C., and the boards $14.80 per M. Required the number of posts, the amount of lumber, and the cost of both.

CAPACITY OF BINS, CISTERNS, ETC.

482. The ***Standard Bushel*** of the United States contains 2150.42 cu. in., and is a cylindrical measure 18½ in. in diameter and 8 in. deep.

1. Measures of capacity are all *cubic* measures, solidity and capacity being measured by different units, as seen in the tables.

2. The *Imperial Bushel* of Great Britain contains 2216.192 cu. in.

3. The *English Quarter* contains 8 Imp. bushels, or 8¼ U. S. bu.

4. Grain is shipped from New York by the *Quarter* of 480 lb. (8 U. S. bu.), or by the *ton* of 33⅓ U. S. bushels.

5. A *Register Ton,* used in measuring the entire internal capacity or *tonnage* of vessels, is 100 cubic feet.

6. A *Shipping Ton,* used in measuring *cargoes,* is 40 cubic feet in the U. S., and in England 42 cubic feet.

7. The bushel *heaped measure* is the Winchester bushel heaped in the form of a cone, which cone must be 19½ in. in diameter, and at least 6 in. high.

8. Grain, seeds, and small fruits are sold by *stricken measure.*

9. Corn in the ear, potatoes, coal, large fruits, coarse vegetables, and other bulky articles are sold by *heaped measure.*

10. It is sufficiently accurate in practice to call 5 *stricken* measures equal to 4 *heaped* measures.

483. To find the *exact capacity* of a bin in bushels.

RULE.—*Divide the contents in cubic inches by* 2150.42; *the quotient will represent the number of bushels.*

484. To find the *cubic* contents in a given number of bushels.

RULE.—1. *Multiply the number of bushels by* 2150.42; *the product will be the number of cubic inches, which may be reduced to higher denominations if required.*

Since a standard bushel contains 2150.42 cu. in., and a cubic foot contains 1728 cu. in., a bu. is to a cu. ft. nearly as 4 to 5; or a bu. is equal to $1\frac{1}{4}$ cu. ft., nearly. Hence for all practical purposes,

2. *Any number of cubic feet diminished by* $\frac{1}{5}$ *will represent an equivalent number of bushels.*

Thus, 250 cu. ft. − $\frac{1}{5}$ of 250 cu. ft., or 50 cu. ft. = 200, the number of bushels in 250 cu. ft.

3. *Any number of bushels increased by* $\frac{1}{4}$, *will represent an equivalent number of cubic feet.*

Thus, 200 bu. + $\frac{1}{4}$ of 200 bu., or 50 bu. = 250, the number of cubic feet in 200 bushels.

WRITTEN EXERCISES.

485. 1. A bin is 6 ft. long, 5 ft. wide, and 4 ft. deep. How many bushels will it hold?

2. A rectangular box will hold 128 bu. What is its volume in cubic feet?

3. How many bushels of wheat can be put in a bin 8 ft. long, 6 ft. 6 in. wide, and 3 ft. 4 in. deep?

4. What must be the depth of a bin to contain 240 bu., its length being 10 ft. and its width 5 ft.?

OPERATION.—240 bu. + 60 bu. = 300; $300 \div \overline{10 \times 5} = 6$ ft., the depth.

RULE.—*Divide the contents in cubic feet or inches by the product of the two dimensions, in the same denomination.*

5. What must be the length of a bin that is 6 ft. wide and 4½ feet deep, to contain 324 bushels?

6. What must be the width of a bin 12 ft. long and 10 ft. deep, to contain 900 bushels of shelled corn?

7. A bin that holds 100.8 bu. is 7 ft. long and 6 ft. deep. How wide is it?

8. How many bushels will fill a bin that is 8.5 ft. long, 4.25 ft. wide, and 3¾ ft. deep?

9. A bin 10 ft. long, 6 ft. wide, and 4 ft. deep, will hold how many bushels of oats? Of potatoes?

10. How many bushels of apples will a wagon-box hold, that is 12 ft. long, 3 ft. wide, and 2 ft. 6 in. deep? How many bushels of barley?

11. A bin 20 ft. long, 12 ft. wide, and 5 ft. deep, is full of wheat. What is its value at $2 a bushel?

12. A bin 7 ft. long, 6 ft. wide, and 5 ft. deep is ¾ full of rye. What is its value at $1.37½ a bushel?

13. A farmer's entire crop of barley just filled a bin 10 ft. long, 6 ft. wide, and 5 ft. deep. What was its value, at $1.78 per cental?

14. A crib, the inside dimensions of which are 15 ft. long, 7 ft. 4 in. wide, and 8 ft. high, is full of corn in the ear. If 2 bu. of ears make 1 bu. of shelled corn, what is the value of the whole, when shelled, at $.92 a bushel?

15. If one bushel or 60 lb. of wheat make 48 lb. of flour, how many barrels of flour can be made from the contents of a bin 10 ft. long, 5 ft. wide, and 4 ft. deep, filled with wheat?

16. How many tons of ice can be packed in a building 40 ft. long, 30 ft. wide, and 20 ft. high, a cubic foot of ice weighing 58⅛ pounds?

17. John Wheatley & Co. bought 12400 bu. of wheat, delivered in New York, at $1.50 a bushel. They shipped the same to Liverpool, paying 6s. sterling per quarter freight, and sold the entire cargo at 12s. per cental. Making no allowance for exchange or for waste, what was the gross gain in U. S. Money?

1. ***Coal.*** Ordinary anthracite coal measures from 36 to 40 cu. ft. to the ton; bituminous coal, from 36 to 45 cu. ft. to the ton.

2. Lehigh, white ash, egg size, measures about 34½ cu. ft. to the ton (2000 lb.); Schuylkill, white ash, 35 cu. ft., and of gray or red ash, 36 cu. ft. to the ton.

5. Coal is bought and sold in large quantities by the ton; in small quantities by the bushel, the conventional rate being 28 bu. (5 pecks) to a ton, or about 43.5 cu. ft.

18. How many tons of red ash coal, egg size, will a bin 17 ft. long, 6 ft. wide, and 3 ft. deep, contain?

19. A bin 6 ft. long, 4 ft. deep, and 5 ft. 9 in. wide is full of Lehigh white ash coal. Find its value at $6.75 a ton?

20. A large crib 10 yd. long, 6 yd. wide, and 6 yd. deep is filled with Schuylkill red ash coal. Find the number of tons it contains, and its value at $5⅛ a ton?

21. A bin 7 ft. long, 5 ft. wide, and 5 ft. deep is half-full of Schuylkill white ash coal. Find its value at $5.90 a ton.

486. The ***Standard Liquid Gallon*** of the United States contains 231 cu. in., and is equal to about 8⅓ lb. Avoir. of pure water.

1. The half-peck, or *dry gallon,* contains 268.8 cubic inches.

2. *Six dry* gallons are equal to nearly *seven liquid* gallons.

3. The *Imperial Gallon* of Great Britain contains 277.274 cu. in., *and is equal to* about 1.2 U. S. Liquid Gallons.

487. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cubic in. in one gallon.	Cubic in. in one quart.	Cubic in. in one pint.	Cubic in. in one gill.
Liquid Measure	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry Measure ($\frac{1}{8}$ pk.) . .	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

A *cubic foot* of pure water weighs 1000 oz., equals $62\frac{1}{2}$ lb. Avoir.

488. To find the *exact* capacity of a vessel or space in gallons:

RULE.—*Divide the contents in cubic inches by* 231 *for liquid gallons, or by* 268.8 *for dry gallons.*

489. To reduce gallons to cubic inches:

RULE.—*Multiply the given number of liquid gallons by* 231; *then reduce to higher denominations if required.*

WRITTEN EXERCISES.

490. 1. How many gallons of water will a cistern hold, that is 4 ft. square and 6 ft. deep?

OPERATION.—$(4 \times 4 \times 6 \times 1728) \div 231 = 718\frac{10}{77}$ gal.

2. How many gallons will a tank 4 ft. long, 3 ft. wide, and 1 ft. 8 in. deep contain?

3. How many barrels of water will a vat hold that contains 43659 cubic inches?

4. How many cubic feet in a space that holds 48 hhd.?

5. How many hogsheads will a cistern 11 ft. long, 6 ft. wide, and 7 ft. deep contain?

6. Find the weight of water in a bath-tub 6 ft. long, 3 ft. wide, and 1 ft. 9 in. deep.

7. How many gallons will a space contain that is 22.5 ft. long, 3.25 ft. wide, and 6.4 ft. deep?

8. A man constructed a cistern to hold 32 hhd., the bottom being 6 ft. by 8 ft. What was its depth?

9. How many more cubic inches in 189.5 gallons dry measure than in 189.5 gallons liquid measure?

10. Find the number of gallons in a cubic foot, correct to 4 decimal places.

11. A tank in the attic of a house is 6 ft. 6 in. long, 4 ft. wide, and 3 ft. 6 in. deep. How many gallons of water will it hold, and what will be its weight?

12. If 64 quarts of water are put into a vessel that will exactly hold 64 quarts of wheat, how much will the vessel lack of being full?

13. If a man buy 10 bu. of chestnuts at $5 a bushel, dry measure, and sell the same at 25 cents a quart, liquid measure, how much does he gain?

14. A cistern 5 ft. by 4 ft. by 3 ft. is full of water. If it are emptied by a pipe in 1 hr. 30 min., how many gallons are discharged through the pipe in a minute?

15. A vat that will hold 5000 gallons of water will hold how many bushels of corn?

16. A tank is 7 ft. deep, 4 yd. long, and 3 yd. broad. What weight of water is in it when just *half-full?*

17. A cellar 40 ft. long, 20 ft. wide, and 8 ft. deep is half-full of water. What will be the cost of pumping it out, at 6 cents a hogshead?

18. A reservoir 24 ft. 8 in. long by 12 ft. 9 in. wide is full of water. How many cubic feet must be drawn off to sink the surface 1 foot? How many gallons?

19. How many imperial gallons will a cistern contain that is 7 ft. 3 in. long, 3 ft. 8 in. deep, and 2 ft. 10 in. wide?

491. The ***Avoirdupois Pound*** contains 7000 Troy grains; hence the Troy pound is $\frac{5760}{7000} = \frac{144}{175}$ of an Avoir. pound; but the Troy ounce contains $\frac{5760}{12} = 480$ grains, and the Avoir. ounce, $\frac{7000}{16} = 437.5$ grains.

The pound, ounce, and grain of Apothecaries' Weight are the same as those of Troy Weight, the *ounce* being *differently divided.*

492. COMPARATIVE TABLE OF WEIGHTS.

		Troy.		Avoirdupois.		Apothecaries'.
1 pound	=	5760 grains	=	7000 grains	=	5760 grains.
1 ounce	=	480 "	=	437.5 "	=	480 "
		175 pounds	=	144 pounds	=	175 pounds.

WRITTEN EXERCISES.

493. 1. Change 10 lb. 8 oz. Avoir. weight to Troy.

OPERATION.—10 lb. 8 oz. = 168 oz.; 168 oz. × 437.5 = 73500 gr.; and 73500 gr. ÷ 480 = 153⅓ oz. = 12 lb. 9⅓ oz. Troy.

2. Change 15 lb. 10 oz. 12 pwt. Troy to Avoirdupois.

OPERATION.—15 lb. 10 oz. 12 pwt. = 190.6 oz.; 190.6 oz. × 480 = 91488 gr.; and 91488 gr. ÷ 437.5 = $209\frac{175}{202}$ oz. = 13 lb. $1\frac{175}{202}$ oz.

3. Find the value in Troy weight, of 9 lb. 10 oz. Avoir.

4. What is the value in Avoirdupois weight, of 16 lb. 8 oz. 10 pwt. 12 gr. Troy?

5. What is the value of a coffee urn, weighing 2 lb. 14 oz. Avoir., at $1.80 per ounce Troy?

6. How many ounces of gold are equal in weight to 6 lb. of lead?

7. If 8 lb. Avoir. of drugs are bought for $12½ a pound, and retailed at the rate of $16.25 a pound, Apothecaries' weight, what is the gain on the whole?

8. What is the difference in the weight of 42⅜ lb. of iron and 42.375 lb. of gold?

494. SYNOPSIS FOR REVIEW.

DENOMINATE NUMBERS—CONTINUED. MEASUREMENTS.

- 1. **RECTANGULAR SURFACES.**
    - 1. Rectangle.
    - 2. Area of Rectangle.
    - 3. Unit of Measure.
    - 4. Rules.
        - 1. *To find area of rect.*
        - 2. *To find either dimen.*
- 2. **ARTIFICERS' WORK.** How computed.
- 3. **GOVERNMENT LANDS.**
    - Range.
    - Township.
    - Sub-divisions.
    - *Table.*
- 4. **RECTANGULAR SOLIDS.**
    - 1. Definitions.
        - 1. *Rect. Solid.*
        - 2. *Volume.*
    - 2. Unit of Measure.
    - 3. Rules.
        - 1. *To find volume of a rectangular solid.*
        - 2. *To find a required dimension.*
- 5. **MASONRY.**
    - 1. How estimated.
    - 2. Rules.
        - 1. *To find number of bricks in a cu. ft. of masonry.*
        - 2. *To find number of Pch. of stone in a given work.*
- 6. **BOARDS AND TIMBER.**
    - 1. Definition of board foot.
    - 2. Rules, 1, 2
- 7. **CAPACITY OF BINS, ETC.**
    - 1. Standard Bushel of U. S.
    - 2. Rules.
        - 1. *To find capacity of bins.*
        - 2. *To find cu. contents of a given number of gal.*
        - 3. *To find either dimen.*
    - 3. Standard Liquid Gallon of U. S.
    - 4. Comp. Table of Meas. of Capacity.
    - 5. Rules.
        - 1. *To find capacity of a vessel in gallons.*
        - 2. *To reduce gal. to cu. in.*
        - 3. *To reduce cu. ft. to bu.*
        - 4. *To reduce bu. to cu. ft.*
        - 5. *To find 1 dimen. of a bin.*

ANSWERS.

The answers to the introductory and more simple examples of many of the articles have been omitted.

Art. 77.

1. $5.98.
2. $39.18.
3. $137.87.
4. $247.78.
5. $38.58.
6. $27.78.
7. $189.75.
8. $17.67.

Art. 79.

2. 1646.
3. 1619.
4. $65.94.
5. $287.67.
6. $376.71.
7. 4491.
8. 7504 lb.
9. 75686.
10. 72447.
11. $696.87.
12. $18.12.
13 $80.87.
14. $149.18.
15. 105233.
16. $220.34.
17. 181776.
18. 11965.
19. $944.65.
20. $7193.28.
21. $3551.05.
22. 1547164.
23. $6692.23.
24. 15873478.
25. $104560.

Art. 91.

10. $14.11.
11. 3231.
12. $51.24.
13. 2123 tons.
14. 2324 ft.
15. 2324 days.
16. $41.23.
17. $230.43.
18. $202.12.
19. 224113.
20. 721220.
21. 210532.
22. 4175.
23. 51.
24. 5113.
25. $15.21.
26. $22.10.
27. $25.26.
28. 2710.
29. 34213.
30. $212.20.
31. $6746.
32. 221533.

Art. 93.

2. 1848.
3. 3833.
4. 1318.
5. 4195.
6. 28286 miles.
7. 26762 acres.
8. 228670 ft.
9. $240.81.
10. $95.58.
11. $38.08.
12. $6.16.
13. 32358.
14. $64.84.
15 $135.28.
16. $157.63.
17. 8728 rd.
18. 45736 tons.
25. 12336.
26. 37588.
27. 69356.
28. 4800.
29. $3323.59.
30. $1264.50.
31. 33798.
32. 35555.
33. $291.35.
34. $222.75.
35. $3015.05.
36. $5524.77.
37. 10386.
38. $695.79.
39. $5351.84.
40. $101.10.
41. 474889.

Art. 95.

1. 332650.
2. $895.66.
3. 30443.
4. 6132.
5. 2517.
6. $15.22.
7. 4190 miles.
8. $3640.
9. 78388 sq. mi.
10. 3572 ft.
11. $53945.
12. $9505.67.
13. 1909609.
14. $5044.25.
15. $16948.50.
16. $1417.16.
17. 702.
18. $36.50.
19. 8346.
20. 16552.

Art. 105.

12. $4743.
13. $1956.
14. $6190.
20. $40.50.
21. $30.59.
22. $622.50.
23. $16120.

Art. 107.

2. 12771.
3. 25830.
4. 34104.
5. $1239.30 ; $1713.15.
6. $3885.75 ; $4521.60.
7. $2209.32 ; $2383.74.
8. 482400 ; 430944 ; 874752.
9. 2953216 ; 5606496 ; 7083104.
10. $85692.24 ; $279759.96 ; $171384.48.
11. $2529.25.
12. $319192.
13. $14064.
14. $264958.
15. 404914.
16. 186516.
17. 241768.
18. $51188.62.
19. 17902976.
20. $154037.36.
21. 15704325 da.
22. 2082600 cts.
23. 1508741097.
24. 1587862270.
25. 3654860576.
26. 8198473608.

27. 982275037.
28. 8363731415.
29. $2715413.50.
30. $21718.16.
31. 416304.
32. 0.
33. 947363302.
34. 5395144320.
35. 72618.
36. $3594.24.
37. $4101.25.
38. 51408 ;
$7454160.
39. 277536 ;
$49956.48.

Art. 109.

2. $3505.92.
3. 3605472.
4. 3906168.
5. $19789.44.
6. 84338.28.
7. 16810320.
8. 54793296.
9. $109804.80.
10. $9212.
11. $430.08.
12. $19234.32.

Art. 110.

5. $472.
6. $1824.
7. $840000.
8. 600000.
9. 12600000.
10. 104000000.
11. 126930871800.
12. 350310024000.
13. 96000.
128000.
268800.
14. $400000.

Art. 113.

1. $1617.30.
2. $50.19.
3. $829.56.
4. $3023.75.
5. 17920.
6. 2878.
7. 37200.
8. 151218.
9. $7198.75.
10. $18801, Whole.
$10938, Farm.
$4617, Stock
11. $8232.
12. $25 loss.
13. 77050.
14. 92500.
15. $1714.50.
16. $43187.32.

Art. 133.

15. 1887 ;
7303 ;
2883.
16. $47208\frac{1}{7}$;
2754 ;
$131181\frac{4}{7}$.
17. $48475\frac{6}{9}$.
$67297\frac{2}{9}$.
$115458\frac{4}{9}$.
18. $172.65.
19. $5801\frac{1}{6}$ lb.
20. $2584\frac{1}{8}$ days.
21. $820.50.
22. $71474\frac{4}{7}$ mi.
23. 8219 men.
24. $20116\frac{1}{3}$ A.
25. 63362 rd.
26. 1592 bbl.
$883\frac{4}{5}$ “
27. 9375 bu.
28. $108.50.
29. 93 oranges.
30. 91 yd.
31. $256.

Art. 136.

3. $2340\frac{2}{14}$;
$2047\frac{10}{16}$;
$1424\frac{10}{28}$;
$1170\frac{2}{28}$.
4. $2248\frac{42}{58}$;
$2070\frac{16}{68}$;
$1610\frac{16}{81}$;
$1762\frac{88}{74}$.

Art. 138.

9. $9.58.
10. $14.89.
11. $25.21.
12. 354 times.
13. 416 “
14. 672 “
15. 1763 “
16. 3300 “
17. $13\frac{20}{105}$ “
18. $4\frac{448}{700}$ “
19. $642\frac{2}{47}$;
$592\frac{41}{47}$;
$2201\frac{27}{47}$.
20. $1083\frac{21}{185}$;
$25414\frac{35}{185}$.
21. $$1823\frac{86}{313}$.
22. $97.
23. $76.
24. 475 acres.
25. 37 horses.
$110 left.
26. 394.
27. 5482.
28. 7198.
29. 31416.
30. 7071.
31. 8723.
32. 610.
33. $28004\frac{1752}{2624}$.
34. $1172\frac{664}{786}$.
35. 4321.
36. 2036.
37. $3645\frac{2867}{3076}$.
38. 7500.
39. 43785.
40. 4629.
41. 346.

Art. 139.

2. 173.
3. 285.
4. 4175.
5. 437.
7. $1931\frac{42}{72}$.
8. $76671\frac{68}{96}$.
9. 4175.
10. $544\frac{82}{135}$.
11. $1640\frac{176}{245}$.

Art. 140.

8. $279\frac{8305}{25000}$.
9. $2824\frac{361}{2040}$.
10. $545\frac{7160}{14800}$.
11. $43.
12. 20 lots.
13. $84\frac{20}{470}$.

Art. 145.

1. 4920.
2. 9 times.
3. 9 “
4. 394950.
5. 538.
6. $443\frac{80}{173}$.
7. $10.78.
8. 16399.
9. $28.15.
10. 3000 lb.
11. $7.50.
12. 42588.
13. 718284.
14. 7 years.
15. 55552.
16. 50496.
17. 7325.
18. 826776.
20. 76 cts.
21. $107.
22. $$2123\frac{3}{8}$.
23. 42 weeks.
24. $367.
25. $1806.
26. $30247.
28. 3823 ;
1849.
29. $720 ;
$530.
30. 2008 ;
1781.

31. \$16550; \$11925.
32. 24 boxes.
33. 356 cords. \$4 cost.
34. 288.
35. 2.
36. 10.
37. 1476.
38. 49.

Art. 165.

2. $2, 3, 5^2, 7$.
3. $2^2, 3, 5, 19$.
4. $3, 5, 163$.
5. $2, 7, 13^2$.
6. $3^2, 5, 7^2$.
7. $2, 3, 5, 7, 11$.
8. $3, 5, 7, 11$.
9. $2, 3^2, 163$.
10. $2^2, 3^2, 5^2, 7$.
11. $3^2, 5, 7^2$.
12. $11, 31, 41$.
13. $2^6, 5, 101$.
14. $2^{10}, 3, 7$.
15. $3^2, 5^2, 7, 19$.
16. $19, 23, 29$.
17. $2, 5, 7, 11, 13$.
18. $3^2, 5, 7^2, 13$.
19. $2, 5, 7, 11, 41$.

Art. 170.

2. 14.
3. 32.
4. 5.
5. 18.
6. 144.
7. 22.
8. 42.
9. 24.

Art. 171.

2. 4.
3. 7.
4. 27.
5. 2.
6. 1.
7. 13.
8. 13.
9. 113.
10. 17; 87.
11. 124; 2.
12. 12 ft.
13. 3 bu.
14. 4329 bags.
15. 5; 9; 11 hr.
16. 8162 rails.

Art. 177.

2. 2856.
3. 120.
4. 450.
5. 30030.
6. 13860.
7. 1680.
8. 5280.

Art. 178.

2. 4896.
3. 16800.
4. 51282.
5. 1560.
6. 7200.
7. 3060.
8. 1680.
9. 315.
10. 240.
11. 180 ft.
12. \$60.
13. 384.
14. 63.
15. \$4536.
16. 720 bu.

Art. 182.

3. 13.
4. 4.
5. 14.
6. 130.
7. 33.
8. 61.
9. 14839.
10. 16.
11. $9\frac{1}{7}$.
12. $1\frac{1}{98}$.
13. 32.
14. 403.
15. $1\frac{3}{7}$.
16. $41\frac{1}{7}$.
17. 10 tons.
18. 98 bbl.
19. $8\frac{1}{3}$ tubs.
20. 16 shillings.
21. \$.50.
22. \$.36.
23. 120 bu.
24. 44 yd., 1st. 22 yd., 2d.
25. \$.77.
26. 144 bu.
27. 4 chests.
28. 48 days.

Art. 209.

13. $\frac{4}{5}$.
14. $\frac{48}{47}$.
15. $\frac{4}{7}$.
16. $\frac{21}{22}$.
17. $\frac{71}{198}$.
18. $\frac{3}{13}$.
19. $\frac{71}{198}$.
20. $\frac{7}{8}$.
21. $\frac{19}{20}$.
22. $\frac{117}{265}$.
23. $\frac{1106}{1665}$.
24. $\frac{28}{794}$.

Art. 211.

3. $\frac{1944}{24}$.
4. $\frac{1248}{16}$.
5. $\frac{800}{7}$.
6. $\frac{337}{20}$; $\frac{488}{20}$.
7. $\frac{3105}{15}$.
8. $\frac{2459}{18}$.
9. $\frac{12279}{26}$.
10. $\frac{21729}{40}$.
11. $\frac{17512}{95}$.
12. $\frac{169287}{84}$.
13. $\frac{4907}{24}$ days.
14. $\frac{39312}{126}$.
15. $\frac{115887}{84}$.
16. $\frac{90557}{90}$.

Art. 213.

5. $17\frac{13}{28}$.
6. $28\frac{3}{5}$.
7. $30\frac{3}{4}$.
8. $18\frac{2}{3}$.
9. $1018\frac{15}{17}$.
10. $50\frac{5}{29}$.
11. $60\frac{1}{3}$.
12. $98\frac{899}{1046}$.
31. $1029\frac{183}{617}$.

Art. 218.

2. $\frac{27}{68}$; $\frac{35}{68}$.
3. $\frac{35}{60}$; $\frac{48}{60}$.
4. $\frac{72}{168}$; $\frac{105}{168}$; $\frac{112}{168}$.
5. $\frac{21}{48}$; $\frac{32}{48}$; $\frac{18}{48}$.
6. $\frac{63}{70}$; $\frac{50}{70}$; $\frac{35}{70}$.
7. $\frac{39}{234}$; $\frac{52}{234}$; $\frac{36}{234}$.
9. $\frac{36}{60}$; $\frac{35}{60}$; $\frac{44}{60}$.
10. $\frac{42}{48}$; $\frac{33}{48}$; $\frac{34}{48}$.
11. $\frac{24}{78}$; $\frac{45}{78}$; $\frac{14}{78}$.
12. $\frac{160}{168}$; $\frac{27}{168}$; $\frac{102}{168}$.
13. $\frac{22}{48}$; $\frac{42}{48}$; $\frac{33}{48}$; $\frac{34}{48}$.
14. $\frac{105}{140}$; $\frac{380}{140}$; $\frac{84}{140}$; $\frac{238}{140}$.
15. $\frac{125}{20}$; $\frac{7}{20}$; $\frac{140}{20}$; $\frac{30}{20}$.
16. $\frac{360}{960}$; $\frac{200}{960}$, $\frac{210}{960}$; $\frac{2688}{960}$.
17. $\frac{114}{168}$; $\frac{67}{168}$; $\frac{336}{168}$; $\frac{24}{168}$.

Art. 221.

3. $1\frac{19}{30}$.
4. $5\frac{39}{40}$.
5. $82\frac{29}{36}$.
6. $262\frac{6}{35}$.
7. $2\frac{5}{1326}$.
8. $9\frac{27}{28}$.
9. $22\frac{17}{70}$.
10. $106\frac{1}{24}$.
11. $69\frac{5}{32}$.
12. $490\frac{13}{28}$.
13. $251\frac{7}{24}$ yd.
14. $9\frac{5}{18}$ yd.
15. $201\frac{47}{48}$.
16. $88\frac{1}{5}$.
17. $191\frac{20}{21}$.

Art. 224.

3. $\frac{7}{18}$.
4. $\frac{13}{28}$.
5. $\frac{331}{840}$.
6. $26\frac{3}{14}$.
7. $56\frac{7}{102}$.
8. $164\frac{29}{48}$.
9. $\frac{31}{88}$.
10. $\frac{11}{12}$.
11. $1\frac{13}{15}$.
12. $12\frac{9}{11}$.
13. $7\frac{25}{72}$.
14. $10\frac{7}{12}$.
15. $\frac{1}{980}$.
16. $16\frac{46}{375}$.
17. $183\frac{17}{27}$.
18. $248\frac{1}{132}$.
19. $115\frac{18}{25}$.

Art. 226.

1. $88\frac{5}{52}$, the greater.
2. $69\frac{20}{63}$.
3. $182\frac{41}{56}$.
4. $1014\frac{11}{63}$.
5. $\frac{5}{48}$.
6 $\frac{23}{63}$.
7. $149\frac{1}{48}$.
8. \$$1\frac{37}{48}$.
9. $1\frac{17}{168}$.
10. $6\frac{3}{22}$.
11. $34\frac{13}{14}$.
12. $68\frac{11}{30}$.
13. $158\frac{13}{27}$.
14. $328\frac{63}{65}$.
15. $265\frac{3}{10}$.
16. $699\frac{9}{20}$.

Art. 229.

2. $7\frac{5}{7}$.
3. $2\frac{3}{7}$.
4. $11\frac{8}{17}$.
5. $1\frac{2}{5}$.
6. $13\frac{1}{2}$.
7. $2\frac{5}{8}$.
8. $\frac{2}{3}$.
9. $16\frac{5}{6}$.
10. 24.
12. 15.
13. $2\frac{2}{3}$.
14. 20.
15. 126.
16. $128\frac{4}{7}$.
17. 255.
18. $5\frac{13}{47}$.
19. 72.
20. $119\frac{1}{3}$.
22. 1532.
23. 1287.
24. 5386.
25. $3949\frac{1}{3}$.
26. 15099.
27. $12756\frac{3}{25}$.
29. 1212.
30. 3624.
31. 7429.
32. 3729.
33. 13272.
34. $10200\frac{2}{3}$.
35. 23586.
$208993\frac{1}{2}$.
322.
36. \$580.
37. \$$15\frac{1}{5}$.
38. \$$1769\frac{1}{16}$.
39. \$$910\frac{3}{4}$.
40. \$$7\frac{1}{2}$; \$$22\frac{1}{2}$; \$60; \$$112\frac{1}{2}$.

Art. 232.

2. $\frac{11}{38}$.
3. $\frac{4}{9}$.
4. $\frac{17}{72}$.
5. $\frac{3}{5}$.
6. $\frac{1}{844}$.
7. $\frac{16}{119}$.
9. $9\frac{181}{225}$.
10. 512.
11. 32.
12. 1530.
13. $62\frac{4}{7}$.
14. 5278.
15. $18\frac{1}{7}$.
16. $\frac{99}{190}$.
17. \$$69\frac{3}{8}$.
18. \$$21\frac{21}{38}$.
19. \$345.
20. \$$12\frac{3}{20}$.
21. \$$63\frac{9}{80}$.
22. \$$145\frac{3}{5}$.
23. \$$23\frac{1}{8}$.
24. \$$11.65\frac{1}{2}$.
25. \$$2293\frac{3}{4}$.
26. \$7196.
27. \$5734.
28. \$$47\frac{17}{24}$.
29. \$$\frac{13}{32}$.
30. \$$28\frac{1}{8}$.
31. \$$199\frac{33}{50}$.
32. \$$73\frac{7}{40}$.
33. \$$110\frac{29}{32}$.
34. $424\frac{46}{81}$.

Art. 235.

2. $\frac{4}{47}$.
3. $\frac{5}{216}$.
4. $\frac{13}{806}$.
5. $\frac{5}{811}$.
6. $\frac{1}{513}$.
7. $\frac{1}{42}$.
9. $6\frac{1}{8}$.
10. $16\frac{11}{160}$.
11. $4\frac{43}{756}$.
12. $5\frac{25}{144}$.
13. $12\frac{77}{800}$.
14. $17\frac{1}{36}$.
15. \$$\frac{1}{12}$.
16. $5\frac{17}{84}$.
17. $\frac{7}{180}$ A.
18. \$$10\frac{19}{48}$.
19. $14\frac{7}{15}$.
20. \$$85\frac{3}{5}$.
21. $39\frac{7}{256}$.
22. $6\frac{27}{32}$ lb.
23. $176\frac{1}{12}$ lb.

Art. 238.

2. 117.
3. 126.
4. $205\frac{5}{7}$.
5. $408\frac{13}{25}$.
6. $877\frac{7}{9}$.
7. $1486\frac{10}{13}$.
8. \$147.
9. $13\frac{3}{17}$.
10. $20\frac{85}{143}$.
11. $29\frac{2}{5}$.
12. $67\frac{7}{19}$.
13. \$$3\frac{1}{5}$.
14. 6 sons.
17. $\frac{6}{7}$.
18. $\frac{3}{4}$.
19. $\frac{56}{81}$.
20. $3\frac{3}{8}$.
21. $1\frac{5}{8}$.
22. $\frac{2}{7}$.
23. 36.
24. $\frac{8}{875}$.
25. $1\frac{5}{7}$.
26. $\frac{36}{55}$.
27. $5\frac{23}{32}$.
28. $17\frac{25}{81}$.
29. $1\frac{1}{4}$.
30. $1\frac{107}{325}$.
31. $1\frac{5}{7}$.
32. $1521\frac{9}{16}$.
33. 63.
34. $8\frac{23}{29}$.
35 \$10450.
36. $12\frac{1}{4}$ tons.
37. \$$\frac{43}{148}$.
38. \$$2.18\frac{1}{4}$.
40. $1\frac{1}{6}$.
41. $1\frac{23}{125}$.
42. $6\frac{43}{81}$.

43. $1\frac{17}{18}$.
44. 2.
45. $\frac{18}{85}$.
46. $6\frac{1}{4}$ mo.
47. $1\frac{189}{350}$.
48. $2\frac{49}{62}$.
49. $\frac{25}{98}$.
50. $15\frac{87}{146}$.
51. $5\frac{78}{185}$.
52. $\frac{747}{3542}$.

Art. 242.

1. $\frac{3}{4}$.
2. $\frac{1}{64}$.
3. $\frac{3}{4}$.
4. $\frac{6}{7}$.
5. $\frac{3}{10}$.
6. $\frac{5}{36}$.
7. $\frac{1}{15}$.
8. $\frac{4}{25}$.
9. $\frac{1}{12}$.
10. $\frac{1}{26}$.
11. $\frac{1}{6}$.
12. $\frac{1}{5}$.
13. $\frac{1}{3}$.
14. $\frac{32}{55}$.
15. $\frac{23}{150}$.
16. $\frac{1}{20}$.

Art. 245.

1. $\frac{20}{72}$, $\frac{48}{72}$, $\frac{63}{72}$,
$\frac{56}{72}$.
2. 84.
3. $1677\frac{17}{40}$.
4. $3\frac{17}{30}$.
5. $3043\frac{1}{8}$.
6. 9072.
7. $4612.
8. $$10588\frac{4}{17}$.
9. $7\frac{7}{12}$.
10. $10946.
11. $4.
12. $.75.
13. $$4577\frac{3}{80}$.
14. $$410\frac{13}{15}$.
15. $4\frac{7}{8}$ tons.
16. $146\frac{2}{23}$ miles.
17. $3\frac{3}{7}$.
18. $13\frac{1}{3}$ days.
19. $$192\frac{13}{16}$.
20. $$4\frac{31}{152}$.
21. $5625.
22. Inc'd $\frac{1}{30}$.
23. Dim'd $\frac{8}{35}$.
24. $9\frac{13}{48}$ bu.
25. $\frac{25}{84}$.
26. $$\frac{1}{2}$.
27. 378 bbl.
28. $$3329\frac{1}{6}$;
47 acres.
29. $108.
30. $14\frac{7}{10}$ cords.
31. 20 bbl.
32. $$1\frac{3}{4}$.
33. $1840.
34. $152\frac{1}{2}$ ft.
35. $1\frac{7}{10}$; $2\frac{1}{2}$.
36. $3224 cotton
$2418 sugar.
$1488 mon'y
$9672 total.
37. $6\frac{169}{1050}$.
38. $\frac{25}{98}$.
39. $5\frac{50}{81}$.
40. $1\frac{16}{75}$.
41. $\frac{611}{660}$.
42. $2\frac{1}{3}$.
43. $22\frac{1}{15}$.
44. $\frac{817}{1628}$.
45. $\frac{1}{2}$.

Art. 267.

18. .596.
19. .0625.
20. .0012.
21. .000074.
22. .0000105.
23. .000099010.
24. .437549.
25. 3040000.-
0000012.
26. 600.00000-
024.
27. 495705000.-
0043075.
28. 4735000.-
00903624.

Art. 283.

15. $$\frac{3}{8}$.
16. $$\frac{5}{8}$.
17. $$\frac{1}{12}$.
18. $\frac{1}{16}$.
19. $\frac{7}{12}$.
20. $\frac{15}{16}$.
21. $$\frac{1}{3}$.
22. $$\frac{2}{3}$.
23. $$\frac{1}{6}$.
24. $\frac{7}{36}$.
25. $\frac{4}{9}$.
26. $\frac{33}{40000}$.
27. $$15\frac{2}{5}$.
28. $$36\frac{3}{4}$.
29. $$9\frac{5}{8}$.
30. $$27\frac{3}{8}$.
31. $24\frac{4}{15}$.
32. $84\frac{1}{18}$.
33. $38\frac{5}{12}$.
34. $104\frac{1}{125}$.

Art. 285.

3. $.75.
4. $.875.
5. .56.
6. .9375.
7. $.8.
8. $.495.
9. .024.
10. .8125.
11. .83333+.
12. .25925+.
13. .76785+.
14. .24666+.
15. .60625.
16. .05078+.
17. .003125.
18. .005625.
19. .7.
20. .032.
21. $1.875.
22. $.066.
23. 101.75.
24. 225.625.
25. 11.125.
26. 8.6625.
27. $.934375.
28. $4.008.
29. 12.69.

Art. 288.

2. 1.703326.
3. 599.007.
4. $206.874.
5. .058815.
6. 51.180606.
7. $275.215.
8. 150.0660325.
9. 79.9992.
10. 111.233 A.
11. $70.03.
12. 1.5547+.
13. $7062.15.
14 387.33 rods.
15. $5984.80.

Art. 290.

2. 253.86319.
3. $533.06.
4. $26.6875.
5. .376118.
6. $161.085.
7. 1.99655.
8. 10.040174.
9. 103.5.
10. 4.9999875.
11. $.25.
12. $0625.
13. 6.3045.
14. .238517+.
15. 1.873125.
16. $129.0625.
17. $.35\frac{3}{8}$.
18. .57675.
19. .09.
20. 2194.85 A.
21. $$6.458\frac{1}{3}$.
22. $411.58.
23. 1.6625.
24. 4.075.
25, $95.
26. $47.07.

Art. 293.

2. .33615.
3. 14.21628.
4. .00087.
5. 24.5470625.
6. $105.138.
7. $36.0062.
8. 572.8.
9. 620.7.
10. 1.375.
11. 676.
12. 20.496.
13. .04765625.
14. .0431388.
15. 7.03125.
16. 15.015.
17. .0084375.
18. 1252.6875.
19. $53.5.
20. 114 75.
21. .0615.
22. $155.8475.
23. 556.718bu. +
24. $446.25.
25. $438.
26. $14891.925.
27. $53.696 + .
28. $12300 75.
29. $113.235 + .
30. $115.237 + .
31. $389.49.
32. $242.937 + .
33. 402.788976.
34. 4.437.
35. 1.69064.
36. 7.03175.

Art. 296.

2. $22.66\frac{2}{3}$.
3. 4500.
4. .2.
5. 1.25.
6. 36.4.
7. 4602.
8. 73.73 ;
24.5766 + ;
5898.4 ;
85.0730 + .
9. $95 ;
$15.125.
10. $14\frac{1}{3}$ times ;
80 "
6.42 + "
11. $.8\frac{2}{3}$; $6.66\frac{2}{3}$;
.075.
12. $3.13133\frac{2}{3}$;
$313.133\frac{2}{3}$;
$3131.33\frac{2}{3}$;
$31313.3\frac{2}{3}$;
$313133\frac{2}{3}$.
13. 387.5 ;
38.75 ;
3.875 ;
.3875.
14. .6455.
15. 50000 times.
16. $8000.
17. .4.
18. .16.
19. .1344.
20. .0175.
21. .00734.
22. $.72.
23. .0001.
24. 100000.
25. 121.875.
26. $.0033\frac{1}{3}$; 10.
27. 23 tons.
28. 12 coats.
29. 17 horses.
30. 136 bbl.
31. $65.406 + .
32. 550 lb.
33. $6.25.
34. 135 lb.
35. 6.375.
36. 1554.
37. 688.
38. 2.07887 + .
39. 11.338 + .
40. $295.

Art. 305.

1. .4375.
2. .8; .875; .36;
.8125 ; .575.
.088 ; .385.
4. $.\dot{8}5714\dot{2}$;
$.\dot{7}$; $.\dot{8}\dot{1}$; $.\dot{3}2\dot{4}$;
$.\dot{4}7619\dot{0}$;
$.\dot{1}707\dot{3}$.
6. $.41\dot{6}$; $.5\dot{3}$;
$.\dot{5}\dot{9}0$; $.3\dot{6}$;
$.31\dot{3}$.
7. .12 ; 125 ;
$\dot{2}941176470$-
$58823\dot{5}$;
484375 ; $.\dot{6}$;
28125 ; .088;
$\dot{2}3809\dot{5}$;
.2288.

Art. 306.

2. $\frac{5}{11}$.
3. $\frac{2}{3}$.
4. $\frac{11}{37}$.
5. $\frac{25}{37}$.
6. $\frac{12}{37}$.
7. $\frac{42}{101}$.
8. $\frac{31}{111}$.
9. $\frac{107}{333}$.
10. $\frac{65}{101}$.
11. $\frac{97}{909}$.
12. $\frac{39}{41}$.
13. $\frac{12}{13}$.
14. $2\frac{11}{37}$.
15. $12\frac{3}{37}$.

Art. 307.

2. $\frac{26}{45}$; $\frac{11}{225}$;
$\frac{113}{1125}$; $\frac{173}{275}$.
3. $7\frac{263}{495}$; $2\frac{559}{990}$;
$7\frac{5}{396}$.
4. $\frac{43}{925}$.
5. $\frac{523}{666}$.
6. $\frac{13}{14}$.
7. $\frac{13}{37}$.
8. $\frac{5}{396}$.
9. $\frac{95}{18}$.

Art. 313.

4. $172.
5. 19.83\frac{1}{3}$.
7. $856.
8. $384.

Art. 315.

2. $157.875.
3. $3986.722
4. 44.83\frac{1}{2}$.
5. $4696.30.
6. $65.875.
7. $438.75.
8. $9.1875.
9. $40.176.
10. $17.71.
11. $325.80 + .
12. $183.15.
13. $212.75.
14. $85.93 + .
15. $23.96 + .

Art. 316.

2. $6.66 + .
3. $30.34 + .
4. $630,70 + .
5. $39.65
6. $20.173 + .

Art. 320.

1. Dr. $812 72.
2. Cr. $21788.16.

Art. 327.

1. $448.07.
2. $1489.46.
3. $1489.84.
4. $6053.50.
5. $81.80.
6. $258.85.
7. Cr. Bal.,
$169.675.
8. Note to Bal.,
$176.16.

Art. 328.

1. $60.
2. $59.57.
3. $21.375.
4. $7.50.
5. $3228.84.
6. $3.40.
7. $.50.
8. $1165.
9. $83.531¼.
10. $191.10.
11. $4.68¾.
12. $122.50.
13. $.33⅓.
14. $.15.
15. $176.475.
16. $104.10.
17. 23 bu.
18. $21.125.
19. 134⅜ tons.
20. 21557.47343.
21. 48 lb. each.
22. $1584, gain.
23. $450.
24. $2.625.
25. $1.25 per C.
26. .15.
27. $4.50.
28. $5.06+.
29. $196.21+
30. 1100 lb.
31. 80 bu.
32. 1.68+.
33. $3.40+.
34. 138 bu.
35. $74, cost. $59, selling price.
36. $232.745.

Art. 425.

2. 45515 gr.
3. 105948 oz.
4. 910 in.
5. 68245 min.
6. 63964 ft.
7. 2046 in.
8. 222 eighths.
9. 43695 sq. ft.
10. 224800 P.
11. 8960 sq. A.
12. 29106 l.
13. 9696 cu. ft.
14. 6216 pt.
15. 792 qt.
16. ʒ 4800.
17. 1008 gi.
18. 14918 lb.
19. 5480 pwt.
20. ℈4785.
21. 525600 min.
22. 7948800 sec.
23. 8784 hr.
24. 19325.
25. 200 quires.
26. 864 doz.
27. 78360 d.
28. 2650 ct.
29. 8280 d.
30. $1045.50.
31. 960 rd.
32. 563 bbl.
33. 80 boxes.
34. $29.25.
35. 13440 times.
36. 1440 min.
37. 2160 sheets.
38. $36.
39. 2419200.
40. 11082240.
41. 417 min.
42. 36 bottles.
43. 876576 hr.
44. 1485 copies.
45. 256 pp.
46. 3625 lb.
47. 8344 lb.
48. 2950 lb.
49. 32620.
50. 6325 lb.
51. 3150 lb. N.Y.
52. 7545 " "
53. 461824 "
54. 1400 lb.
55. 1800 lb.
56. $136.262.
57. $124.095+.
58. $4.825.
59. $10.13⅝.

Art. 428.

2. 15 w. 4 da. 9 hr. 40 min.
3. 10 mi. 8 ch. 20 l.
4. 2031 lb. 9 oz. 10 pwt.
5. 50 mi.
6. 1605 A.
7. 1 sq. mi.
8. 125 cu. ft. 840 cu. in.
9. 297 C. 26 cu. ft.
10. 15 hhd. 19 gal. 3 qt. 1 pt.
11. 846 bu.
12. 264 bbl. 26 gal. 3 qt.
13. 12965 gal.
14. Cong. 63, O. 2, ℥ 10.
15. 14 lb. 10 oz. 18 pwt. 22 gr.
16. 25 T. 15 cwt. 70 lb.
17. 25 cwt. 37 lb. 15 oz.
18. 12 lb. 6 oz.
19. 201 bu.
20. 15½ bbl.
21. 208 bu.
22. 31.72 quin.
23. 5 w. 1 da. 1 hr. 1 min. 1 sec.
24. 191 mo. 8 da. 11 hr. 40 min.
25. 557° 33′ 20″.
26. 87 deg. 50 naut. mi.
27. 836 gro. 1 doz. 4 pens.
28. 227⅚ doz
29. 251 sc.
30. 22 Rm. 7 Qu. 10 sh.
31. 151 Bund. 8 Qu.
32. 411 Cr. 2s.
33. 2038 fl.
34. 80 half-sov.
35. £44 2s. 2d. 2 far.
36. 450 fr.
37. 46 sov. 6s. 3.9+d.
38. 200 marks.
39. $1689600.
40. 4725 lb.
41. $32.55 N. Y.
42. 7° 3′.
43. 456 da. 12 hr. 45 min.
44. $1.87½.
45. $48.
46. $56.16.

Art. 431.

2. $\frac{1}{8}$ ft.
3. $\frac{1}{2}$ gr.
4. $\frac{3}{4}$ pt.
5. .24s.
6. $\frac{3}{80}$ oz.
7. $\frac{7}{9}$ sq. rd.
8. .32 pt.
9. $1\frac{5}{7}$ yd.
10. .33 ft.
11. $\frac{8}{9}$ oz.
12. $\frac{5}{6}$ yd.
13. $\frac{5}{9}$ l.
14. .252 min.
15. $\frac{54}{325}$ sq. rd.

Art. 433.

2. 10s. 10d.
3. ℥4 ʒ1 ℈1 gr. 16.
4. 85 rd. 5 ft. 6 in.
5. 9 oz.
6. 3 ft. 9 in.
7. 8.8 oz.
8. 17 da. $3\frac{3}{7}$ hr.
9. 11s. 1.2d.
10. 86 P. 4 sq. yd. 2 sq. ft. $60\frac{11}{13}$ sq. in.
11. $6\frac{2}{3}$ oz. Avoir.
12. 14 cu. ft. $691\frac{1}{5}$ cu. in.
13. 5° 48′ 7.2″.
14. f ʒ5 ♏ 36.
15. 18 cwt. 96 lb. 14 oz.
16. 55 gal. 1 pt.
17. 16 sq. yd. 7 sq. ft. 36 sq. in.
18. ℥8 ʒ1 ℈1 gr. $7\frac{3}{11}$.
19. 6 gro. $10\frac{2}{7}$ doz.
20. 2 mi. 106 rd. 1 ft. 10.176 in.
21. 3 gal. 3 qt. 1 pt. 2 gi.
22. 2 pk. 2 qt. 1 pt.
23. 212 rd.
24. 4 T. 5 cwt. $55\frac{5}{9}$ lb.
25. 1 A. 60 P.
26. 2 Cd. 89.6 cu. ft.
27. 57 rd. 9 ft. $10\frac{1}{5}$ in.
28. 6 Qu. 6 sheets.
29. 54.16\frac{2}{3}$.

Art. 435.

2. $\frac{1}{44}$ gal.
3. £$\frac{1}{90}$.
4. .01 bu.
5. .0001 lb.
6. .0004 ton.
.018 ton.
7. $\frac{3}{98}$ cord.
8. .00045 oz.
9. $\frac{1}{2800}$ ton.
10. $\frac{1}{640}$ da.; .005 da.
11. .02 rd.
12. $\frac{12}{85}$ pt. less.
13. $\frac{7}{1440}$ A.

Art. 437.

2. $\frac{1}{8}$ bbl.
3. .4375 Cd.
4. .22+ hhd.
5. $\frac{569}{640}$ lb.
6. $\frac{1}{8}$.
7. .100304.
8. ʒ.7.
9. .005489 Tp.
10. .3.
11. $\frac{23}{42}$.
12. .57758 lea.
13. $\frac{3}{16}$.
14. .001625.
15. .09.
16. $\frac{25}{36}$.
17. £$\frac{12}{13}$.

Art. 438.

1. 44352 steps.
2. 18 h. 45 min.
3. $53.1665125.
4. $199.25.
5. $111.94.
6. £60.
7. 128.23\frac{1}{3}$.
8. $34.574.
9. $708\frac{3}{13}$ bu., Ill.
$657\frac{4}{7}$ bu., La.
$634\frac{26}{29}$ bu., N. Y.
10. $5714\frac{2}{7}$ bu., Ct.
$5333\frac{1}{3}$ bu., N. J.
11. $332.679.
12. 15 carats.
13. 424.98775 A.
14. $90.
15. $13455.09+.
16. $3.525.
17. $87.815.
18. $2.46+.
19. $\frac{7}{9}$.
20. $138.95.
21. 3125 bu.
22. 720 centals.
23. 5 bbl. 152 lb.
24. $21.988+.
25. $\frac{1723}{4784}$.
26. $2.93.
27. $487\frac{1}{2}$ Rm.
28. $306\frac{1}{4}$ Rm.

Art. 440.

3. 22 yd. 2 ft. 10 in.
4. 19 Cd. 3 cd. ft. 13 cu. ft.
5. 2 hhd. 17 gal. 2 qt. 3 gi.
6. 15 h. 28 min.
7. 60 gal. 1 qt.
8. 22 cwt. 84 lb. $14\frac{2}{7}$ oz.
9. 10 Pch. $9\frac{23}{40}$ cu. ft.
10. ℔ 11 ʒ2 ℈2 gr. 5.
11. $133.2475.
12. $61.50.
13. 31 yr. 11 mo. 3 da.

Art. 441.

3. 25 Cd. 6 cd. ft. 4 cu. ft.
4. 239 rd. 11 ft.
5. 8 cwt. 41 lb. 10 oz.
6. 10s. $7\frac{1}{5}$d.
7. 5 lb. 3 oz. 10 pwt.
8. 1 w. 6 da. 5 hr. 17 min. 16.8 sec.
9. 29 gal. 1 qt. 1 gi.
10. 1 mi. 198.4 rd.

11. $6\frac{5}{8}$ doz.
12. ℥3 ʒ5 ℈2.
13. 20 h. 24 min.
14. 65.44 P.
15. 8 gal. 3 qt.
16. $7306.71+.
17. 13 cwt. 38 lb.
18. 30 Cd. 5 cd. ft. 14 cu. ft.

Art. 442.

2. 7 yr. 9 mo. 1 da.
3. 3 yr. 11 mo. 28 da.
4. 2 yr. 5 mo. 24 da.
5. 258 da.
6. 1 yr. 10 mo. 12 da. 6 h.
8. 204 da.
9. 157 da. 21 h.
10. 2 yr. 5 mo. 8 da. 9 h. 22 min.
11. 2 yr. 8 mo. 10 da. 4 hr. 14 min. 59 sec.

Art. 443.

2. 26 bu. 1 pk. 6 qt.
3. 39 Cd. 3 cd. ft.
4. 13 hhd. 42 gal. 3 qt.
5. 2715 bu.
6. Cong. 25 O. 6 ℥11 ʒ5 ♏36.
7. £120 18s. 6d.
8. 189 A. 40 P. 16 sq. yd. 6 sq. ft.
10. 89 T. 11 cwt. 1 qr. 19 lb. 14 oz.
11. 557 yd. 2 ft. $11\frac{1}{2}$ in.
12. 13 T. 3 cwt. 67.85 lb.
13. $196.796.
14. $125.25.

Art. 444.

2. 51 A. 31 P. 8 sq. ft.
3. £7 1s. 11d.
£5 1s. $4\frac{2}{7}$d.
£4 8s. $8\frac{2}{3}$d.
4. 31 bu. 1 pk. 5 qt. 1 pt.
28 bu. 1 pk. 1.9 pt.
23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt.
5. 12 yd. $5\frac{55}{68}$ in.
6 yd. $2\frac{59}{63}$ in.
6. 2 Cd. 5 cd. ft. $13\frac{1}{2}$ cu. ft.
7. 8580 rails.
8. 1 sq. mi. 42 A. 112 P. 26 sq. yd. 8 sq. ft.
9. 337 yd. 1 ft. $7\frac{1}{4}$ in.
10. 70 times.
11. 243 boxes.
13. 9 cwt. 42 lb.
14. 165 A. 25 P. 24.4 sq. yd,, nearly.

Art. 448.

2. 107° 19′ $48\frac{3}{4}$″.
3. 122° 26′ 45″ W.
4. 71° 12′ 15″ W.
5. 90th W.; 90th E.; 180th E.

Art. 450.

2. 50 min. $21\frac{1}{3}$ sec.
3. 1 hr. 17 min. 24 sec. A.M., next day, or at night.
4. 5 hr. 57 min. 49 sec.
5. 5 h. 59 min. 51 sec.
6. 1 hr. 3 min. 58 sec.
7. 1 h. 13 min. $32\frac{2}{3}$ sec.
8. 51 min. 18 sec.
9. 1 hr. 40 min. 8 sec.
10. 6 hr. 28 min. 27 sec.
11. 1 h. 33 min. 27 sec.
12. 5 hr. 6 min. $15\frac{14}{15}$ sec. A.M. at Cinn.
4 hr. 53 min. 43 sec. A.M. at Chi.
4 h. 43 min. 13 sec. A.M. at St. Louis.
13. 12 h. 7 min. 41 sec., at night, B.
4 h. 53 min. $46\frac{1}{3}$ sec. P.M., St. P.
2 h. 58 min. 6 sec. P.M., Ast. Or.
14. 5 h. 46 min., later, Rome
5 h. 5 min. 32 sec., later, Paris.
15. 10 h. 58 min. 37 sec., loses.

Art. 454.

1. 52 ft. 9′.
2. 319 ft. 4′ 3″.
3. 23 ft. 10′ 9″.

Art. 456.

2. 68 ft.
3. 55 ft. 10′ 3″ 2‴ 8⁗.
4. 240 ft. 9′ 4″.
5. 50 ft. 9′ 10″ 6‴.

Art. 465.

1. 63 sq. yd.
2. $7\frac{1}{2}$ ft. wide.
3. 61 ft. long.
4. 152 sq. yd. 1 sq. ft.
5. 348 sq. yd. 4 sq. ft.
6. $379\frac{1}{4}$ sq. rd.
7. 32 sq. ch. 2 P.
8. $427\frac{1}{8}$ sq. ft.
9. 7 sq. rd. 1 sq. yd. 6 sq. ft. 88 sq. in.
10. 18 ft. 3 in. width.
11. 7 ch. 35 l. length.
12. 18 yd. 2 ft.
13. $58\frac{31}{36}$ sq. yd.
14. 48.
15. 44 yd.
16. 260 yd.
17. $88\frac{2}{3}$ yd.
18. $81\frac{3}{4}$ yd.

19. $98.54+.
20. $106.48.
21. $81.87+.
22. $146.40.
23. $60.95.
24. 1080 tiles.
25. $608.40.
26. $65.475.
27. $21.29\frac{11}{12}$ squares.
28. $139.57.
29. $198.
30. $27.378+.
31. 840 sods.
32. $28\frac{4}{5}$ yd.
33. $13\frac{2}{9}$ rolls.
34. $71.60.
35. $13.525+.
36. 11316 shingles.
37. $25.55.
38. $447.989+.

Art. 467.

1. 90 A.
2. 32 rd. wide.
3. $190\frac{2}{3}$ farms.
4. .625 A.
5. 264 rd.
6. $7594.80+.
7. $\frac{40}{49}$.
8. 25 rd.
9. $220 less.
10. $4000 gain.

Art. 468.

1. 80 A.; $\frac{1}{8}$ Sec.
2. 5760 rails;
$230.40.
3. $340.
4. 240 A.; $\frac{3}{8}$ Sec.
5. 120 A. left;
$27.20 gain.
6. 420 A. left;
$635 gain.

Art. 474.

1. 96 cu. ft.
2. 108 cu. ft.
3. $8\frac{1}{2}$ ft.
4. 221 cu. ft.
5. 208 cu. yd.
6. 3 cu. yd. 26 cu. ft.
297 cu. in.
7. 7 cu. yd. 11 cu. ft.
200 cu. in.
8. 5 cu. yd. 25 cu. ft.
9. $1\frac{1}{8}$ in., height.
10. 8 in., height.
11. 9 ft. 2 in., length.
12. 4840 cu. ft.
13. $12\frac{3}{16}$ Cd.
14. 8 ft.
15. $13.182+.
16. $166.60.
17. 8 ft.
18. 75 cans.
19. $410.156+
20. 24 ft.

Art. 477.

2. $31278\frac{6}{11}$ bricks.
3. 60 Pch.
4. $49\frac{1}{11}$ Pch.
5. 63961+ bricks.
6. $1607.82+.
7. $505.50.
8. 672 Perches.
9. $3276.
10. $423.529+.
11. 2214 cu. ft.;
$344.40 cost.

Art. 481.

3. $53\frac{1}{3}$. *12.* $6.14+
4. $93\frac{1}{2}$. *13.* $1.064.
6. $4.20. *15.* $13\frac{1}{2}$ ft.
7. $15.75. *16.* $62\frac{2}{9}$ ft.
9. 6 ft. *17.* $9.90.
11. 16 in. *18.* $9.425.
19. 315 board ft;
$26\frac{1}{4}$ cu. ft.
20. $159.365.
21. $90.
22. 396 posts;
11880 ft. lumber.
$274.824, cost.

Art. 485.

1. 96 bu.
2. 160 cu. ft.
3. $138\frac{2}{3}$ bu.
5. 15 ft.
6. $9\frac{3}{8}$ ft.
7. 3 ft.
8. $108\frac{3}{8}$ bu.
9. 192 bu. oats;
$153\frac{3}{5}$ bu. potatoes.
10. $57\frac{3}{5}$ bu. apples;
72 bu. barley.
11. $1920.
12. $173.25.
13. $205.056.
14. $259,072.
15. $39\frac{9}{49}$ bbl.
16. $697\frac{1}{2}$ tons.
17. $861.13+.
18. $8\frac{1}{2}$ tons.
19. $27.
20. 270 tons.
$1485.
21. $14.75.

Art. 490.

2. $149\frac{47}{77}$ gal.
3. 6 bbl.
4. $404\frac{1}{4}$ cu. ft.
5. $54\frac{6}{7}$ hhd.
6. $1968\frac{3}{4}$ lb.
7. $3500\frac{68}{77}$ gal.
8. 5 ft. $7\frac{3}{8}$ in.
9. 7163.1 cu. in.
10. 7.4805+ gal.
11. $680\frac{8}{11}$ gal.
$5687\frac{1}{2}$ lb.
12. 604.8 cu. in.
13. $43.09+ gain.
14. $4\frac{76}{77}$ gal.
15. $537\frac{7}{64}$ bu.
16. 23625 lb.
17. $22.797+.
18. $314\frac{1}{2}$ cu. ft.
$2352\frac{48}{77}$ gal.
19. 469.39+ Im. Gal.

Art. 493.

3. 11 lb. 8 oz. 7 pwt
7 gr.
4. 13 lb. 12+ oz.
5. $75.46875.
6. $87\frac{1}{2}$ oz.
7. $57.986+.
8. 52545 gr.